

# Aerodynamics for Professional Pilots

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## Introduction

*The subject we are studying is called aerodynamics, which means that it has to do with “air” and motion. You may find it productive to imagine how an aeroplane would act in the absence of air and then realize that the difference between that and what happens in the presence of air is what we are studying. In the absence of air an aeroplane would simply fall toward the center of the earth. If it was upside down when dropped it would remain so, if dropped tail first it would fall in that orientation, etc. If you spun it, say like a Frisbee, it would spin around its CG, but otherwise fall. If you started it pitching it would also rotate around its CG while falling, and so on. Such motions are called tumbling.*

*In the air an aeroplane does not fall or tumble. In air, aeroplanes tend to fly straight and always with the front forward and the tail behind. At air shows you may have seen aeroplanes slide tail first, but only for a second or two, then they right themselves and fly nose first. The study of aerodynamics must explain all this as well as the obvious topic of lift, and the unfortunate one of drag.*

*This text was written specifically for students in the Professional Pilot Program at Selkirk College. It is tailored to students interested in becoming pilots, not engineers. The objective of this text is to help pilots understand how an airplane flies in order to pilot it more effectively. There is some math involved but no complex calculus or algebra. Many graphs are used to present the relationship between two factors, for example velocity and lift. When faced with a graph it is important to examine it to see whether the relationship between the elements is linear, exponential or otherwise. But it is also important to consider what other variables are involved and what assumptions have been made about their values when the graph was plotted. In the case of velocity and lift for example angle of attack, and air density must remain constant in order for a graph of lift vs. velocity to have meaning. But when flying an airplane a pilot would not normally keep angle of attack constant and there-in lies the dilemma of sorting out the interplay of factors involved. The only solution to this is considerable time and effort spent thinking through the interplay of factors.*

*Another form of mathematics used in several key parts of this text is trigonometry. The relationship between angles and the sides of a triangle, especially right-angle triangles is exploited frequently to deduce relationships, such as g-force and bank in a turn, and thrust and drag in a climb. If you are rusty on your basic trigonometry it would be wise to review it before reading this text.*

*The other mathematical form used in this text is vector diagrams. It is important to be aware of the method of adding vectors by drawing them tip-to-tail. The same technique is used in my text Navigation for Professional Pilots for adding the vectors TAS and wind to obtain GS and TMG. In this text we will frequently add the forces lift, thrust, drag and weight. By adding these vectors we will discover many things about the performance of an airplane.*

*Advanced knowledge of physics is not required to understand this text, but it is assumed that the reader has studied High School physics and is familiar with Newton's Laws as well as basic concepts in physics such as moments, i.e. torque. More complex topics such as energy and work are touched on but are not critical to understand aerodynamics.*

*I hope you enjoy this text. More important I hope you become fascinated by aerodynamics. It is undeniably true that you can fly an airplane without knowing any aerodynamics (lots of pilots do.) But knowing how and why things work can make you a more efficient pilot, and if you take it far enough it will make you a more accurate pilot. In an extreme case it could save your bacon; more likely it will just make you feel better about your professionalism.*

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## Aerodynamics

The word aerodynamics means *air in motion*. Aerodynamics is important to many human pursuits from automobile design to ski racing. In this book we will consider aerodynamics as it affects the design and flight of aeroplanes.

The overall objective of this book is to help you understand how your aeroplane flies in order to fly it better. The discussion is intended for pilots, not engineers. The mathematics used is simple. As much as possible I will try to explain aerodynamics using visual imagery rather than equations and graphs, although some of those will be needed. In order to keep things easy to relate to the familiar units of pounds, feet, and knots rather than kilograms and meters are used throughout.

Aerodynamics is a branch of physics, and as you know physics attempts to explain everything from billiard shots to nuclear explosions. Humans have sought to understand physics since before recorded history, but it was not until Isaac Newton that a coherent explanation arose. Since Newton's time the science of physics has expanded to include the theory of electromagnetism, the theory of Relativity, and Quantum Mechanics. The good news for us is that these additions are not required in aerodynamics. Everything you need to know and understand aerodynamics is contained in what physicists call Newtonian mechanics.

Unfortunately "simple" Newtonian mechanics is not so simple. The greatest minds in history (Plato, Aristotle, and everyone else before Newton) failed to grasp Newton's laws. So we should not be surprised that even today it is not natural to see the world as Newton explained it to us. As citizens of the 21<sup>st</sup> century we are all taught Newton's laws; even so I will take considerable effort in the following pages to guide your understanding of physics from an *academic* form into a *visceral* form. Sometimes people actually find it painful to let go of their instinctive beliefs about how the world works, but if you will take this journey with me you will reach an understanding of how your aeroplane flies and will be able to answer the obligatory Transport Canada exam questions to boot; most important of all you will become a better pilot.

## The "Rules" of Physics

Lots of pilots don't like "intellectual" subjects like physics. All the equations and obscure terminology is a turn off. This book is written for such pilots and as a result takes a straight forward approach that will allow you to grasp what is important to piloting and forget the rest.

To understand aerodynamics you must accept the following six *rules of physics*:

1. A body at rest will remain at rest. A body in motion will remain in motion.
2. Force equals mass times acceleration ( $F=ma$ )
3. For every action there is an equal but opposite reaction.
4. Most forces are pushing forces, NOT pulling.

5. Static air pressure ( $P_s$ ) is proportional to air temperature ( $T$ ) and air density ( $\rho$ )  
 $P_s \propto \rho T$
6. Energy cannot be created or destroyed. Therefore kinetic energy plus potential energy equals total energy ( $KE + PE = TE$ )

We will now go over each of the above rules and begin the journey from academic to visceral understanding. (Remember that we are working in a purely Newtonian frame of reference so nuclear forces, which are part of quantum mechanics, are not relevant.)

### ***Rule 1 – Newton’s First Law***

A body at rest will remain at rest. A body in motion will remain in motion.

Rule 1 is known as Newton’s first law. According to this law the objects around you won’t suddenly start moving, at least not without a force being applied (see rule 2.) The rule also says that if an object is currently moving it won’t stop moving. Keep in mind that in physics an object can be anything from an aeroplane to an air molecule. Here are some examples:

- A classic example of rule 1 is rolling a marble across a smooth surface. The marble rolls on and on, hardly slowing at all, until it hits an obstruction.
- Although you can’t see them, air molecules are in perpetual motion.
- It is important to realize that rule 1 applies to rotational motion as well. So for example the earth is rotating at one revolution per day and will continue to do so forever (if there is no friction.)

Newton’s brilliance was in recognizing the universality of this law. The examples given above of perpetual motion are however far from dominant in “real life.” For example if you take your foot off the gas pedal of your car going down the highway, contrary to Newton’s first law your car slows down. And if you are pushing a shovel of snow along your driveway no one is going to convince you that it will just keep moving if you stop pushing. So it is not surprising that despite our academic understanding of this law *viscerally* we feel as though “you must push on things to get them to move.” (That characterization was doctrine prior to Newton.) Intellectually we realize that friction is the culprit, i.e. we must provide just enough force to offset friction when keeping an object moving. But in the case of an aeroplane in flight how much friction is there? It turns out that for rotation motions such as roll, pitch, and yaw there is almost no friction. We will return to that matter later.

### ***Rule 2 – Newton’s Second Law***

Force equals mass times acceleration ( $F=ma$ )

Rule 2 is known as Newton's second law. It tells us that a force must be applied to an object in order to accelerate it. Acceleration means change in velocity.

Velocity is a vector quantity that specifies both speed and direction. In aerodynamics velocity has magnitude equal to true airspeed and a direction given by heading plus vertical speed. Newton's second law says that a force is needed to change any of airspeed, heading, or vertical speed. The converse is also true, i.e. if airspeed, heading, and vertical speed are all constant there is no net force acting on the aeroplane.

As always, this is easy to grasp intellectually, but when you are "holding the nose up" by pulling back on the control column it is viscerally hard to believe that you are not applying a force. Even so, you may be "really" applying a force to the control column, but not to the aeroplane. It depends on whether airspeed, vertical speed, and heading are constant.

### ***Rule 3 – Newton's Third Law***

Rule 3: For every action there is an equal but opposite reaction.

Rule 3 is known as Newton's law of reactions and is sometimes called the third law..

The sun is pulling on the earth right now with a total force equal to the weight of the planet. But consequently the earth pulls on the sun with an equivalent force. There is no way to avoid this.

The propeller on an aeroplane pushes air back with a certain force. As a result it is pushed forward with an equivalent force, which we call thrust.

### ***Rule 4 – the Nature of Forces***

Rule 4: Most forces are pushing forces, NOT pulling.

Almost all forces, including lift and drag are caused by two objects pushing on each other (air and the aeroplane in the case of lift and drag.) Such forces require direct physical contact. There is no way for an air molecule to grasp an aeroplane and pull on it. But an air molecule can "bang into" an aeroplane and **push** on it.

Most forces require direct physical contact between the object providing the force and the one receiving it. For example a bat must hit a ball to provide a force. Similarly the air must be in contact with an aeroplane to make a force on it.

There are **three** forces in the universe that can *act at a distance* these are gravity, magnetism, and electric attraction; these forces able to *pull* objects toward each other.

Of the three "special" forces gravity is the one that is important in aerodynamics because it is the force we must overcome to get off the ground. Gravity causes an attraction

between the aeroplane and the earth and it constantly pulls the aeroplane toward the center of the earth. As mentioned above, gravity does not require physical contact between the aeroplane and the earth, so it continues even when an aeroplane is in flight. Newton discovered that the force of gravity decreases with distance from the center of the earth, but consider this; we are more than 24,000,000 feet from the center of the earth when at sea level, climbing to a typical flight altitude such as 40,000 feet represents a change of  $1/600^{\text{th}}$ , which is not significant. Pilots may therefore think of gravity as constant, i.e. your aeroplane weighs almost the same amount at 40,000 feet as it does at sea level.

The force of gravity acting on an aeroplane is called its weight (W.) It is given by the equation  $W = mg$ , where  $m$  is mass and  $g$  is acceleration due to gravity. (For weight (W) in pounds mass must be in units called slugs and  $g$  equals  $32.2 \text{ ft/sec}^2$ .)

Let us now return to the question of whether or not forces always act by *pushing* on objects. I have stated that objects **cannot** pull on each other. Does this mean that you can push your car if you run out of gas but you can't pull it? Is that true? Actually it is. To "pull" your car you must tie (or hook) a cable onto your car so that part of the rope or hook pushes some part of the car.

It is a convention of language that when you grasp something and move it toward yourself you are "pulling" it. But in reality you must wrap your hand around it and apply a pressure to some part of the object, in effect you must push it. You cannot pull on it in the way gravity pulls on the moon or the North Pole pulls on a compass needle. Think of as many examples as you want and you will never find one in which you or any other object ever really pull another object save through electric, magnetic, or gravitational attractions.

Pilots must therefore accept that air above an aeroplane cannot pull the aeroplane up. It must be the air below the aeroplane that pushes it up. So the common explanation that a thing called a "low-pressure area" pulls (or "sucks") the aeroplane up into the air clearly cannot be true. Air can *push an aeroplane*, but it cannot pull an aeroplane.

Is the above claim bothering you? Have you decided that this book is flawed? Please don't give up yet, all will be explained.

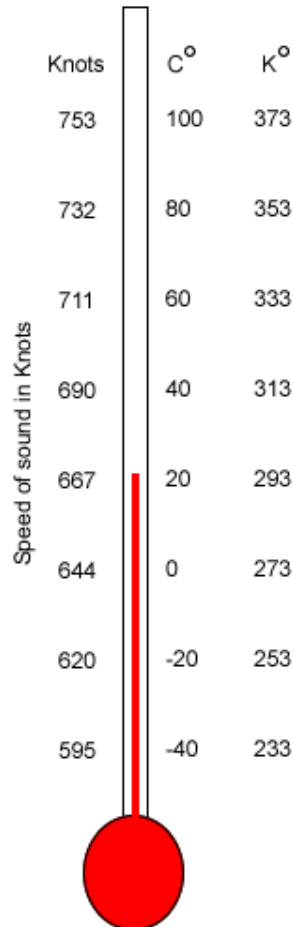
You probably have experience working with vacuum cleaners and you may believe they "suck things up," i.e. pull things. In fact you might have thought of the example of drinking a soft drink with a straw as a situation that appears to violate rule 4. But it actually doesn't. We need to take a look at how a vacuum cleaner actually works, but we can't do that until we examine rule 5.

### ***Rule 5 – The Gas Law***

Static air pressure ( $P_s$ ) is proportional to air temperature ( $T$ ) and air density ( $\rho$ .)

$$P_s = 3089.54 \rho T \quad [R = 3089.54 \text{ if pressure in lb/ft}^2, T \text{ in } ^\circ\text{K}, \rho \text{ in slugs/ft}^3]$$

**A thermometer with an unusual scale**



Air temperature (T) is a measure of the average speed of air molecules. In aerodynamics temperature is measured in units called degrees-Kelvin. In Kelvin, if temperature is above zero then air molecules are in *continuous random motion*, on average at the “speed of sound.” Only if air temperature drops to absolute zero (0°K = -273.16°C) do air molecules stop moving.

Figure 1 shows that a thermometer could be labeled with the speed of sound as a temperature scale. Imagine coming in from outside and saying “its almost 700 knots out there, turn on the air conditioning.”

**Figure 1**

Air density is a measure of the mass of air molecules in a given volume of air. Air density is measured in units of slugs per cubic foot. (1 slug weighs 32.2 pounds on earth.)

Static pressure is a consequence of the air’s weight. Imagine a column of air, one foot by one foot at its base, extending from the earth’s surface to the outer edge of space. This column of air weighs about 2116.16 pounds on a standard day. Static air pressure is therefore 2116.16 lb/ft<sup>2</sup>. The static pressure at an altitude other than sea level is simply the weight of the column of air starting at that altitude and extending upward to the edge of space. A table of these values is given on page 13.

Because air is a gas capable of spreading in all directions and because air molecules resist being compressed, static pressure doesn’t just push down; it pushes laterally and upward as well. Static pressure not only pushes down on the floor of the room you are in it also

pushes on the walls and the ceiling. The upward force on a ten foot by ten foot ceiling is more than 200,000 pounds. That's enough to support a Boeing 737. It's also enough to accelerate the room and everything in it upwards as though it was atop a rocket. Luckily that doesn't happen because an equal 200,000-pound force is pushing down on the top of the ceiling.

An alternate way of thinking about static pressure, and one worthwhile for pilots, is that it is due to the collisions of individual air molecules with objects. I assume you are sitting in a warm room (i.e. well above absolute zero) that contains air (i.e. density is more than zero), and even though the air probably seems motionless it is important to know that the air molecules are rushing around on average at the speed of sound, bouncing off each other, and the walls, floor, ceiling, and you. You don't realize the individual air molecules are moving because each movement is random and only travels a short distance before it bounces off another molecule or *an object*. As the air molecules collide with the walls, floor, and ceiling they *push* outward, as per rule 3. Even though the force of each collision is miniscule there are trillions upon trillions of air molecules striking the sides of the room each second so the net force is substantial.

The gas law quoted above gives the relationship between pressure, density, and temperature under all conditions. However, a special case is very important in aerodynamics, and that is the relationship between pressure, density, and temperature when no outside energy is added to or taken away from the gas (air.) The fancy name for this is isentropic conditions, but it just means that no energy is added or removed. Air flowing past an aeroplane in flight is essentially isentropic. The isentropic relationship for gas is:

$$\left(\frac{P_{s1}}{P_{s2}}\right) = \left(\frac{\rho_1}{\rho_2}\right)^{1.4} = \left(\frac{T_1}{T_2}\right)^{3.5}$$

## Static Pressure Experiments

To grasp the magnitude of static pressure perform some of the following experiments:

1. Purchase a suction cup, with a hook attached, (from any hardware store) and *press* it onto the ceiling. Be sure the chosen “ceiling” is a smooth horizontal surface. The suction cup works by “squeezing out” air to create a vacuum. As you now know, it is the air pushing on the outer surface that actually provides the holding force. A visceral sense of the strength of Ps can be gained by pulling on the suction cup with ones fingers. Compare the size of the suction cup to the wings on an aeroplane. Think about how much force the small suction cup can create and imagine how much a wing could create.
2. Obtain two pieces of glass (it is best if one piece is smaller than the other.) Use a few drops of water to act as an air seal and then push the flat faces of the pieces together. All air will be displaced from between the glass pieces, so that Ps on the outer surfaces holds them together. The smaller piece can be suspended from the larger one. It will float around “frictionless” but not fall. Try to pull it off; it is VERY difficult to do so.
3. Obtain a jar with a sealed lid (e.g. a Mason jar.) Make a hole through the lid for a straw and seal around the straw so that air cannot enter the jar. Put some water in the jar and try to drink it. You will not be able to.

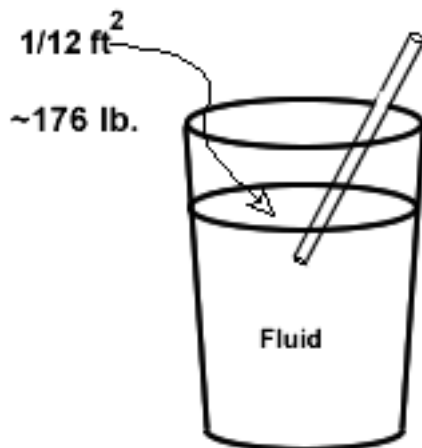


Figure 2

Consider Figure 2. The open surface of the soft drink is 12 square inches ( $1/12^{\text{th}}$  square foot) so a force of ~176 pounds is pushing down on the fluid. That's a lot of force; much more than enough to lift the fluid, which weighs less than one pound, into the drinker's mouth. All the drinker must do is lower the pressure in the straw slightly and the pressure in the room will *push* the fluid into his/her mouth.

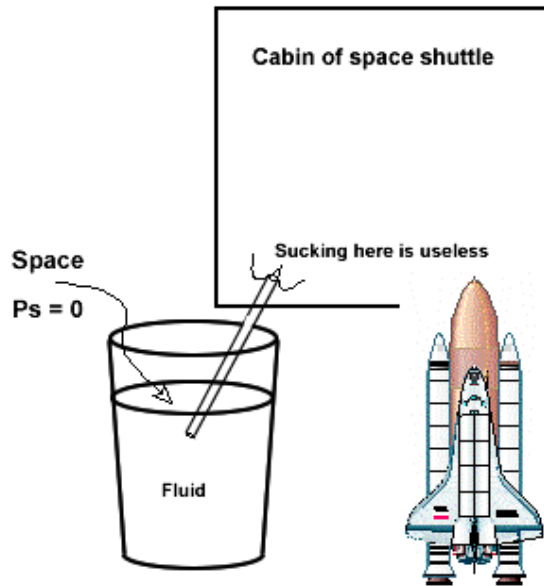


Figure 3

Now consider Figure 3 in which a soft drink glass is floating outside the international space station, where we may say the air pressure is zero. A straw has been installed through the wall and an astronaut is attempting to have a drink. No matter how hard s/he sucks s/he will have no success in getting any fluid. (This effect is replicated with experiment 3 above. In that experiment you cannot drink the water because air pressure drops in the space above the water and is insufficient to push the water up the straw.)

So how does a vacuum cleaner work? Start by contemplating the air in the room where you are sitting. I assume there is no wind in the room. Imagine you have the superhuman ability to see individual air molecules and you are examining a small cluster of molecules somewhere in the room. You would see that each second many molecules in the group leave at high speed, but are replaced, randomly, by others coming in from the surrounding space. No net change results, and since the motions are random the overall air motion is zero. Now consider Figure 4 in which the nozzle of a vacuum cleaner is inserted into the room. What the vacuum cleaner does is remove air molecules from one location in the room. The word *vacuum* means there are fewer air molecules in a given location (and therefore a lower static pressure.) If with your magic eyes you are watching that evacuated space you would notice that since there are fewer molecules in the vacuum then fewer leave each second. But the normal number would randomly enter the area from the surrounding air (which now has a higher static pressure.) As a result there is net motion of air to “fill the vacuum.” You may have heard the expression that “nature abhors a vacuum.” What this old saying means is that the random motions of air molecules will result in air moving from an area of high static pressure to one of low static pressure. When the air molecules rush toward the vacuum they *push* along any dirt



particles they *strike*. It may be common to say we are sucking up the dirt, but as you can see the dirt is really *pushed* along by the air current that forms due to the vacuum<sup>1</sup>.

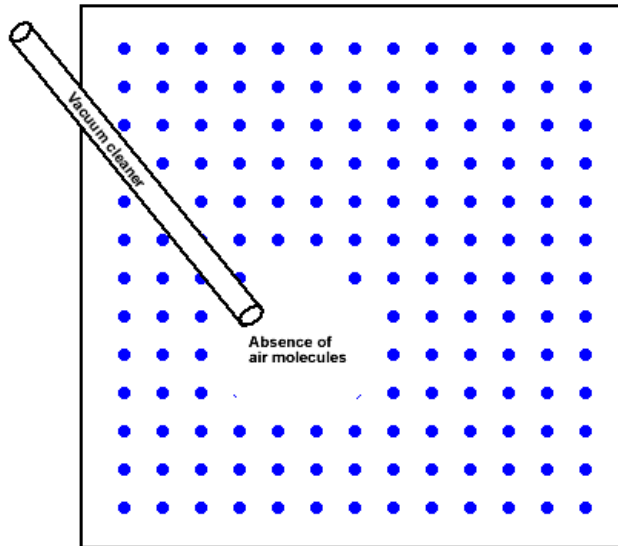


Figure 4

### ***Rule 6 – Conservation of Mass and Energy***

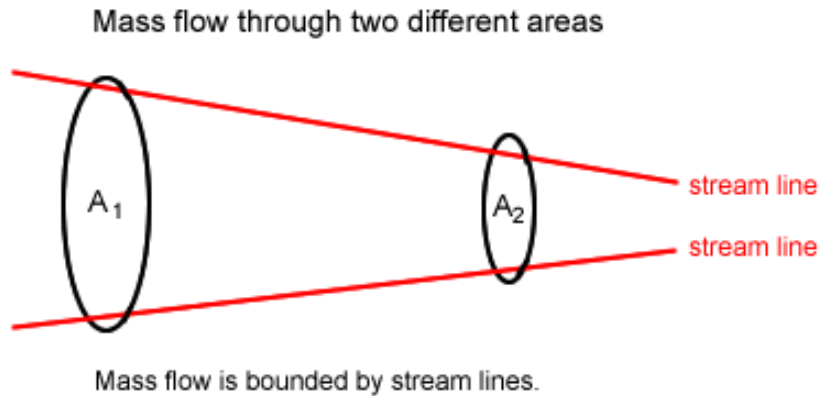
Rule 6 is the expression of two crucial conservation laws:

1. Mass cannot be created or destroyed. This is known as the law of conservation of mass.
2. Energy cannot be created or destroyed. This rule is known as the first law of thermodynamics.

As noted at the beginning of this text aerodynamics is about the motion of air. The conservation of mass law is important because it explains why the velocity of air flowing around a wing changes. Consider Figure 5 which shows air flowing through two circles of different area, labeled  $A_1$  and  $A_2$ .

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<sup>1</sup> You may be asking whether the air “rushes in” at the speed of sound. It actually tries to, but due to collisions between molecules is slowed to a much lower speed. The stronger the vacuum the fewer collisions will occur and the faster the air will move. The ultimate vacuum would cause the air to rush in at the speed of sound.



**Figure 5**

The red lines are called stream lines, and they represent the path taken by air molecules. Imagine that the same stream lines pass through  $A_1$  and  $A_2$ . The situation is analogous to water entering a garden hose at a tap. All the water that enters the hose must flow out the other end. If the hose is pinched at any point along its length the water must still flow through (because matter cannot be destroyed) and therefore must either accelerate or become denser. Water is not easily compressed so it accelerates, as anyone who has pinched the end of a water hose knows. The relevant equation is:

$$A_1 V_1 = A_2 V_2 \quad [\text{Continuity equation for incompressible flow}]$$

$A_1$  represents area 1, and  $A_2$  represents area 2.  $V_1$  and  $V_2$  are the velocity at area 1 and 2 respectively. The equation simply says that at  $A_2$  velocity will be higher if area is smaller.

While the above equation seems clearly correct for water is it valid for air also? Air seems to have a third option; it could compress, i.e. become denser, rather than accelerate. In actuality, as long as air is moving well below the speed of sound the natural forces of repulsion between air molecules keeps density constant, so that the above equation is valid for aerodynamics up to about 1/3 the speed of sound. Once compression becomes significant we must use the equation:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad [\text{Continuity equation for compressible flow}]$$

This equation indicates that the product of density, area, and velocity remains constant. When we study high speed flight we will need to take this into account, but for the first part of the text in which we study subsonic flight we can ignore it and use the incompressible flow *continuity equation*.

Study the two continuity equation above because we will use them to explain how a wing accelerates air, thus creating lift.

We now turn our attention to conservation of energy. You probably remember your high school physics teacher saying that potential energy plus kinetic energy is constant in a closed system. An example is a bouncing ball. Potential energy increases as it rises, while kinetic energy decreases. Kinetic increases as it falls, while potential decreases. Of course a bouncing ball is not a perfectly closed system so energy is lost in the form heat, to the air and the floor.

Air flowing past an aeroplane is essentially a *closed system* and it has both potential and kinetic energies. The potential energy of air could be determined, in principle, just like that of a ball. Potential energy is calculated by multiplying weight and height. In principle the weight of the trillions of air molecules in the atmosphere multiplied by their individual altitudes could be added up to get the potential energy of the atmosphere. This is a hopeless task, but not really necessary. A simple measure of air pressure represents the weight of the atmosphere, and can be used as a proxy for potential energy. We will use the designation  $P_s$  to represent this *static air pressure*, which represents the potential of the atmosphere to do work, i.e. its potential energy. The units of static pressure will be pounds per square foot. This is the same as saying energy per cubic foot.

In physics kinetic energy is  $\frac{1}{2}mV^2$ . Since density is mass per cubic foot the kinetic energy of one cubic foot of air is given by  $\frac{1}{2}\rho V^2$ . The units are also pounds per square foot and therefore we will call this value *dynamic pressure* to distinguish it from *static pressure*.

### **Bernoulli's Equation (The Incompressible-Flow Energy Equation)**

An extremely important relationship between static and dynamic pressure is known as Bernoulli's equation. It is valid for the special case in which density is constant, which we have already said applies to flight at less than 1/3 the speed of sound.

$$P_{s_1} + \frac{1}{2}\rho V_1^2 = P_{s_2} + \frac{1}{2}\rho V_2^2 \quad [\text{Bernoulli's Equation - Metric version}]$$

$$P_{s_1} + 1.426\rho V_1^2 = P_{s_2} + 1.426\rho V_2^2 \quad [\text{Bernoulli's Equation - English version}]$$

In Bernoulli's equation  $1.426\rho V^2$  is called the dynamic pressure (air density must be in units of slugs per cubic foot, velocity in units of knots and thus dynamic pressure is in units of pounds per square foot).

In words Bernoulli's equation is:

*Static pressure plus dynamic pressure equals total pressure.*

## Bernoulli's Equation Experiments

To develop a feel for Bernoulli's equation perform the following two experiments:

1. Hold two pieces of paper, one in each hand and suspend them in front of your face and parallel to each other about four inches apart. Blow gently between them. The increased velocity of the air between the papers reduces  $P_s$ . The higher  $P_s$  on the outer surfaces pushes the two pieces of paper together. This proves Bernoulli's equation is true.
2. Take one piece of paper and dangle it in front of your mouth. It may be better to roll the edge of the paper around a cylinder such as a pencil. Blow over the top surface of the paper. The paper will rise up, just like the wing on an aeroplane.

Armed with Bernoulli's equation, developed from rule 6, and the other five rules, we can now explain how wings produce lift and explore all the other fascinating questions that make up the study of aerodynamics.

## The Compressible-Flow Energy Equation

Bernoulli's equation assumes that density is constant. As we have already said this is reasonable up to about 1/3 the speed of sound. Above that speed we need to use the compressible flow equation, which recognizes compression.

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad [\text{Continuity equation for compressible flow}]$$

When air is compressed not only density but also pressure and temperature change in accordance with the gas law stated previously. Fortunately the isentropic equation applies because energy is not added or removed from the air flow:

$$\left(\frac{P_{s1}}{P_{s2}}\right) = \left(\frac{\rho_1}{\rho_2}\right)^{1.4} = \left(\frac{T_1}{T_2}\right)^{3.5}$$

Combining the equations we get:

$$3800.8 T_1 + 1.426\rho V_1^2 = 3800.8 T_2 + 1.426\rho V_2^2$$

## ***Properties of the Atmosphere***

Before we jump completely into the exploration of aerodynamics we will conduct two brief reviews of material it is assumed you are familiar with from your previous studies. First we will review the important properties of the atmosphere. Second we will review aerodynamic terminology.

The atmosphere is an envelope of gasses that surrounds the earth. It is composed of nitrogen, oxygen, carbon dioxide, water vapor, and other gasses. The most important characteristics, from an aerodynamics point of view, are *temperature*, *density* and *static pressure*.

In the study of aerodynamics we use a model of the atmosphere called the international standard atmosphere (ISA.) The ISA is a temperature model based on observations taken in Paris France. In the ISA sea level temperature is 15°C, which is 288.16°K. The atmosphere is divided into isothermal layers (constant temperature) and gradient layers that have a constant lapse rate. Only the first two layers, known as the troposphere and stratosphere are considered in this course. The troposphere extends from sea level to 36,090 feet with a lapse rate of -1.98°K per thousand feet. The resulting temperatures can be seen in table 1 below. The stratosphere starts at 36,100 feet and extends to approximately 82,000 feet. The stratosphere is isothermal at a temperature of 216.66°K (-56.5°C.)

Once the temperature model is established air density and pressure follow automatically under the influence of gravity<sup>2</sup>. Physicists can calculate what the density of air would be at any given altitude using only the temperature as an input. The equation for density in the troposphere is:

$$\rho = 0.002377(T / 288.2)^{4.2566} \quad [T \text{ in } ^\circ\text{K}, \rho \text{ in slugs/ft}^3] - \text{equation valid up to 36,090'}$$

In the stratosphere the equation for air density is:

$$\rho = 0.000706 e^{-(\text{Altitude} - 36,000) / 20,833} \quad [e \text{ is the natural log base } 2.71828]$$

Once the temperature and density of any gas are known static pressure can be calculated using the gas law. The equation, with the correct unit conversion factor is:

$$P_s = 3089.54\rho T \quad [\text{gives } P_s \text{ in units of lb/ft}^2]$$

---

<sup>2</sup> This is true because the chemical composition of the air does not change throughout the atmosphere

## Aerodynamics for Professional Pilots

Air pressure and density decrease with altitude according to table 1 below.

The speed of sound (a) depends *only* on air temperature (T, in units of degrees Kelvin) and is given by the formula:  $a = 38.98 \times \sqrt{T}$  (a in knots.)

Standard Alt	Temp °K	Static Air Pressure (Ps) Lb/ft <sup>2</sup>	Air Density (ρ) Slugs/ft <sup>3</sup>	Speed of Sound (a) Knots
Sea level	288.16	2116.2	0.002377	661.7
5000	278.26	1760.9	0.002048	650.3
10,000	268.36	1455.6	0.001756	638.6
15,000	258.46	1194.6	0.001496	626.7
20,000	248.56	972.9	0.001267	614.6
30,000	228.76	628.9	0.000890	589.5
40,000	216.66	390.6	0.000584	573.8
50,000	216.66	242.6	0.000362	573.8
60,000	216.66	150.7	0.000225	573.8
100,000	232.66	23.1	0.000032	595.2

Table 1

## Aerodynamic Terminology and Definitions

Every aeroplane has three axes, which you can see in Figure 6. The axes all pass through the center of gravity (defined below.)

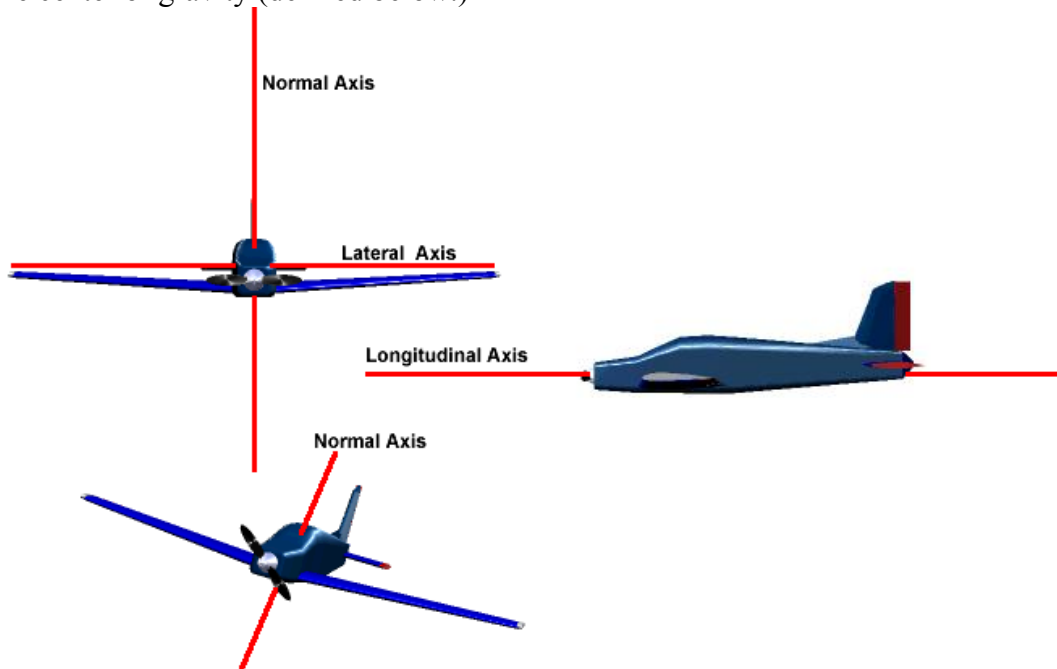


Figure 6

Figure 7 shows the parts of several aeroplane configurations. The **fuselage** is the body of the aeroplane. The wings protrude from the sides. An aeroplane with two wings is called a bi-plane; monoplanes have one wing and are much more common. All the aeroplanes shown below are monoplanes.

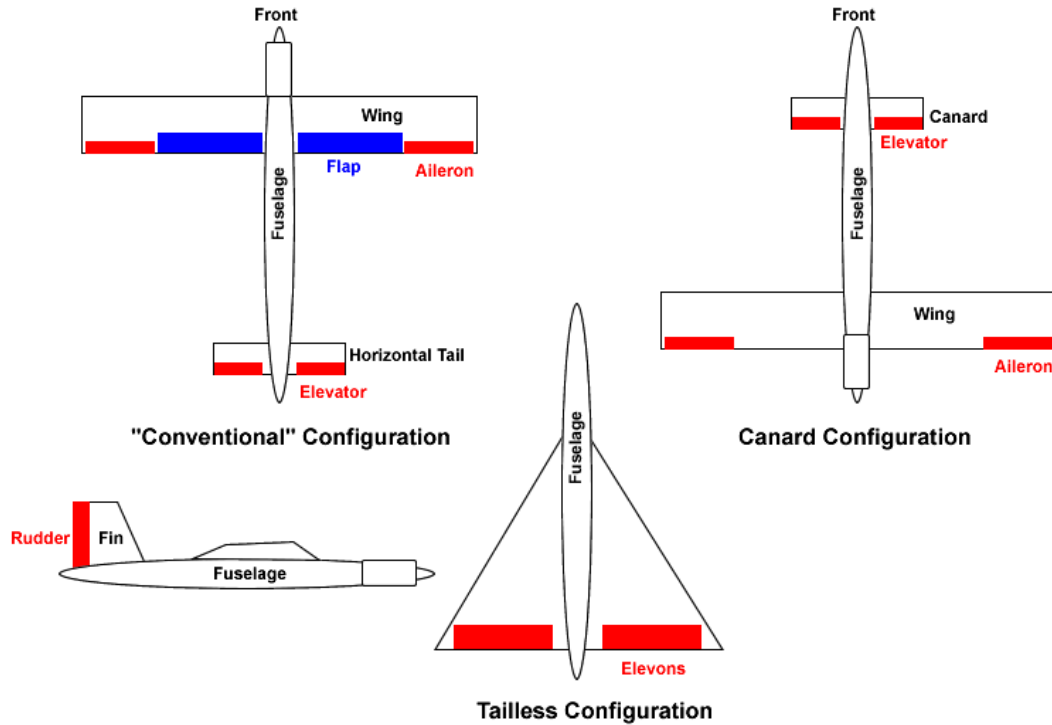


Figure 7

The **ailerons** are almost always attached to the outboard trailing edge of the wing and the **flaps** are on the inboard trailing edge.

The **elevators** are mounted on the trailing edge of the **horizontal** tail, unless the aeroplane is a **Canard**, which means the horizontal tail is near the front. A tailless aeroplane may have separate ailerons and elevators or they may be combined into a combined set of **elevons**.

The **fin** is the non-moving vertical wing, near the rear of the aeroplane; it usually protrudes straight up but can protrude down and can also protrude on an angle. The moveable control attached to its trailing edge is called the **rudder**.

The **horizon** is the flat plane that is tangent to the earth's surface, as shown in Figure 8.





Figure 8

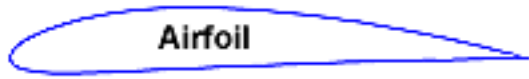
**Pitch attitude** expresses the angle between the longitudinal axis and the horizon plane. If the longitudinal axis is parallel to the horizon we say the aeroplane has zero degrees of pitch. If the longitudinal axis is inclined upward we have positive pitch, and downward is negative pitch. In all cases the pitch attitude is expressed in units of degrees.

**Bank attitude** ( $\beta$ ) expresses the angle between the lateral axis and the horizon plane. If the lateral axis is parallel to the horizon we say bank is zero. Bank angles are expressed as left or right, in units of degrees. Normal flight is limited to less than 90 degrees of bank; however it is possible to talk about larger bank angles such as 180 degrees of bank, meaning that the aeroplane is upside down.

**Heading** is the third “attitude.” It expresses the angle between the longitudinal axis and a reference line in the horizon plane. Normally pilots use either magnetic north or true north as the reference line, but other reference systems could be used. In this book we only need to talk about changes in heading, which means rotation of the longitudinal axis of the aeroplane in the horizon plane. A change in heading is called a **turn**.

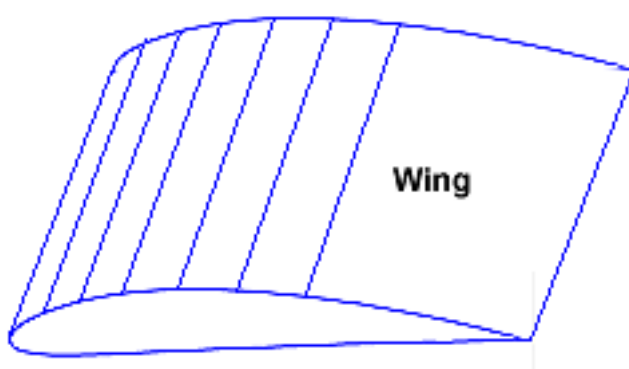
It is important to realize that an aeroplane can make six distinct types of movements. Movement is always referenced to the axes of the aeroplane, not the horizon. The first three movements are translations, the last three rotations. The movements are:

1. Translation along the longitudinal axis, which is the major component of true airspeed
2. Translation along the normal axis, which is the minor component of true airspeed.
3. Translation along the lateral axis, which results in slipping.
4. Rotation around the longitudinal axis, which is called roll.
5. Rotation around the normal axis, which is called yaw.
6. Rotation around the lateral axis, which is called pitch.



**Figure 9**

Consider Figure 9, which is a drawing of an **airfoil**. A drawing is a two-dimensional thing that exists only on a piece of paper or other flat surface. Now imagine the airfoil is stretched out into a three-dimensional wing, as in Figure 10. In this text, when I talk of a **wing** I mean a three-dimensional structure that you would find on an aeroplane. When I talk about an airfoil I am referring to a shape, which exists only in two dimensions.



**Figure 10**

Consider the airfoil in Figure 11. A line can be drawn that is exactly halfway between the upper and lower surface of the airfoil. This line is properly called the **mean-line**, but sometimes is called the camber line.



**Figure 11**

A straight line can be drawn joining the ends of the mean-line and this is called the **chord** line, or simply the chord (c.)

An airfoil may need to be scaled larger or smaller without changing its shape. It is therefore useful to express airfoil dimensions, such as thickness and camber, as a percentage of the chord length, the full length being 100%.

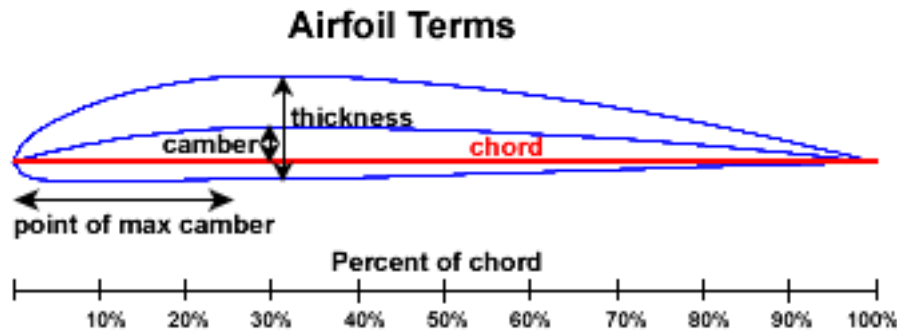


Figure 12

The **thickness** of an airfoil is the maximum distance between the lower and upper surface and is expressed as a percentage of the chord length. In Figure 12 the thickness is 12%<sup>3</sup>.

The *maximum* distance between the chord and mean-line is called the **camber** of an airfoil. Camber is expressed as a percentage of the chord. In the figure camber is 4%, which means the maximum distance between the chord and mean lines is 4% of the length of the chord. Positive camber means the mean line is above the chord line, negative camber would mean the mean line is below the chord. If the mean-line and chord are the same there is zero camber. Such an airfoil is said to be symmetrical, as shown in Figure 13. Symmetric airfoils are widely used for fins and horizontal tails. They are also used on some aerobatic aeroplanes for the main wing.

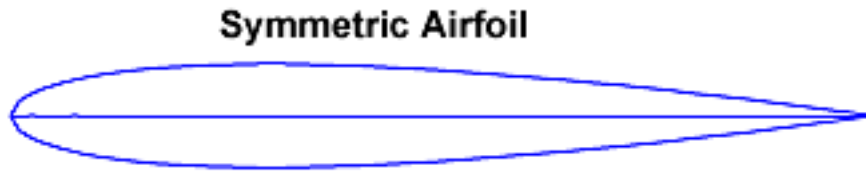


Figure 13

If the mean line crosses the chord the airfoil is called **reflexed**. Figure 14 shows an exaggerated reflexed airfoil.

<sup>3</sup> Strictly the distance should be measured perpendicular to the mean line.

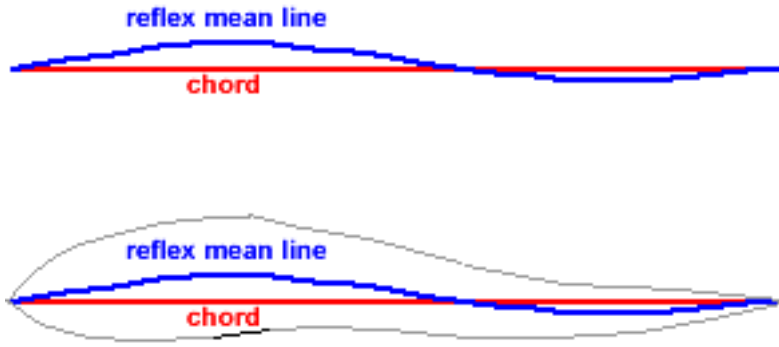


Figure 14

The point along the chord where maximum camber occurs (see Figure 12) is called the **point of maximum camber**. It is at 30% of  $c$  in all the airfoils shown above.

In the days before computer aided aircraft design airfoil shapes were catalogued using a numbering system that specified the thickness, camber and point of maximum camber. For example a C-172 has a 4412 airfoil, which means it has 4% camber with a maximum camber point at 40% and maximum thickness of 12%. Those interested in learning more about airfoil numbering systems should read the excellent NASA publication Theory of Wing Sections<sup>4</sup>.

Modern *wings* are computer designed to work in three dimensions and in unison with the fuselage. In almost all modern aeroplanes the airfoil changes from root to tip of the wing. (root and tip are defined below.)

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<sup>4</sup> Abbot, I. Dover Publications Inc. New York. 1959

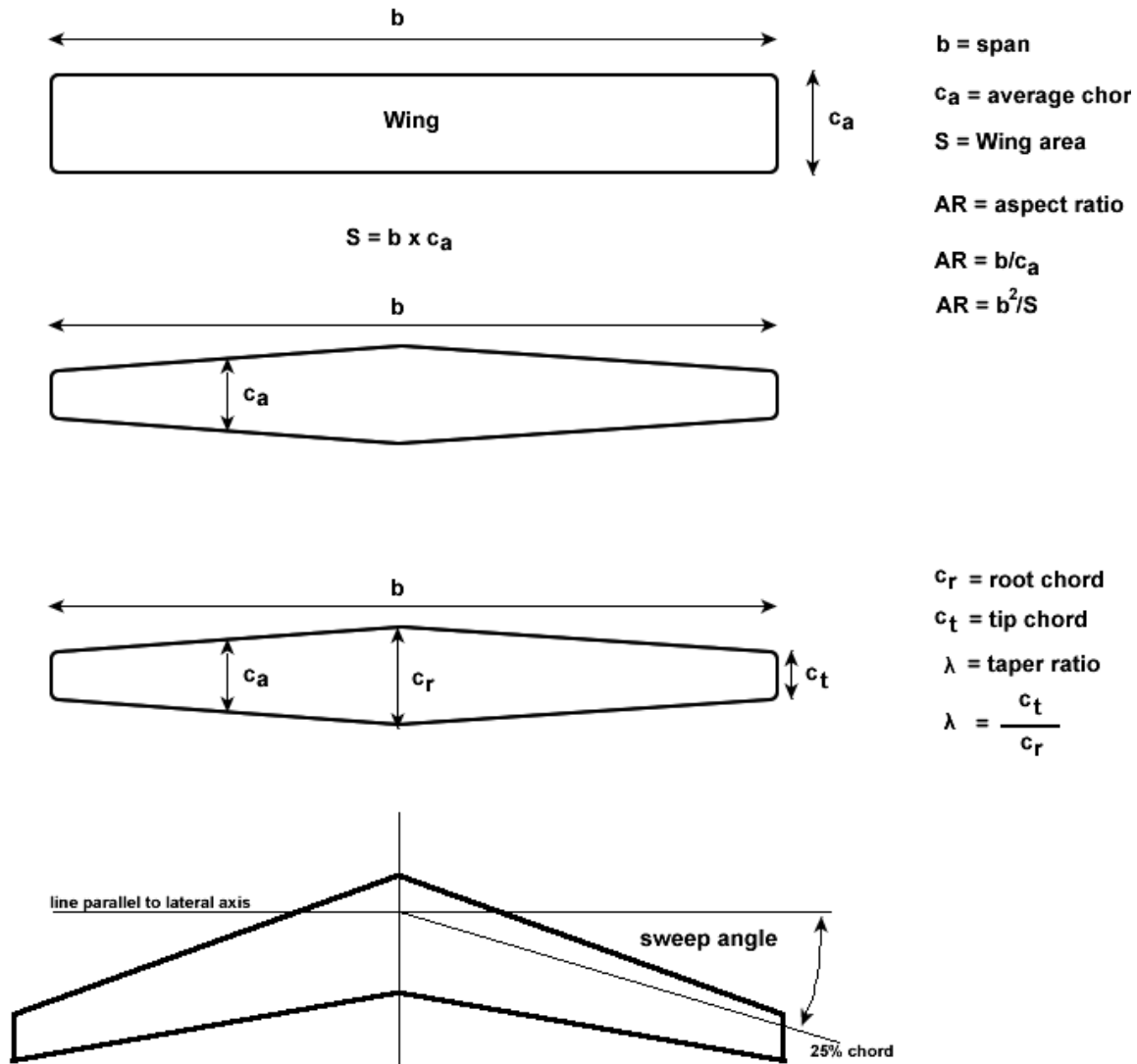


Figure 15

Figure 15 defines the terms **wing root** ( $C_r$ ), **wing tip** ( $C_t$ ), and **span** ( $b$ ). **Taper ratio** is the ratio of  $C_t/C_r$ . The aeroplane in the figure has a taper ratio of 0.5

**Wing area** ( $S$ ) is one of the most important design parameters of an aeroplane. It is the surface area of the wing and is equal to average chord ( $c_a$ ) times span ( $S = c_a \times b$ ).

Another very important design parameter is **aspect ratio** ( $AR$ ), which is the ratio of span to average chord ( $AR = b/c_a$ .) Note that  $c_a$  is the average chord defined as  $c_a = S/b$ . Large aspect ratios are found on gliders and transport jets. Low aspect ratios are found on jet fighters.

Given that  $AR$  and  $S$  are the two most important wing design parameters it is useful to relate them to each other. Therefore if we substitute the definition  $S = c_a \times b$  into the  $AR$

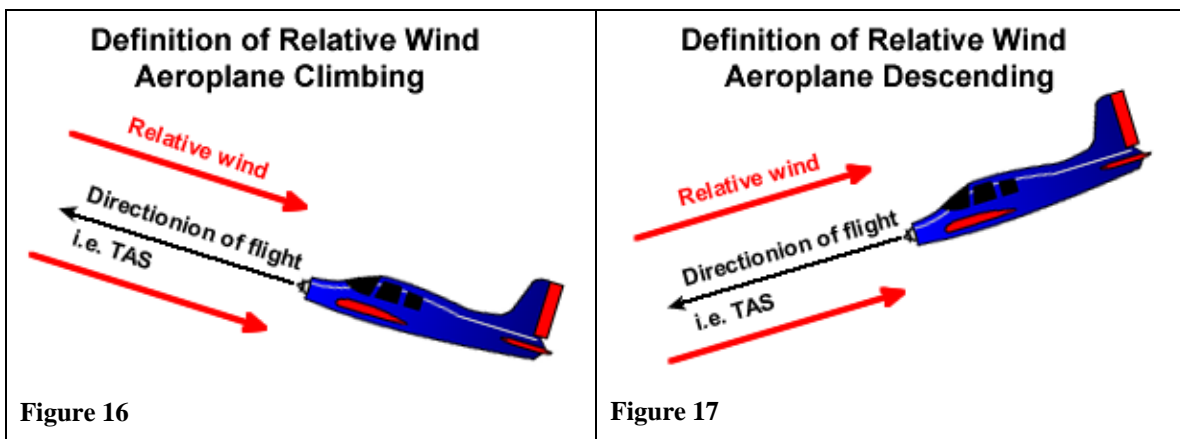
formula we get  $AR = b^2/S$ . Please note that this is not an alternate definition, only a mathematical formulation that is often useful in aeroplane design. You will always find it easier to visualize aspect ratio as  $b/c_a$ .

**Wing sweep** is defined as the angle between a line drawn through the 25% chord points on the wing and the lateral axis, expressed in degrees (see the diagram above.)

**Velocity** (V) is the combination of true airspeed (TAS), heading, and climb angle. It expresses how fast the aeroplane is traveling and the direction it is traveling. Velocity is a vector quantity, which means that you can draw an arrow, to describe what direction it acts. Notice that TAS alone is not a vector quantity because an aeroplane can fly at a constant TAS while turning, or entering a climb or descent etc. The heading and vertical speed values transform the scalar airspeed value into a vector.

We will use terms such as straight, level, climbing, descending, and turning to describe what is happening to velocity. These terms have the following meanings:

- **Straight:** Means flight on a constant heading.
- **Level:** Means flight at a constant altitude. This is synonymous with vertical speed equals zero.
- **Climbing:** Means flight with constant airspeed and constant positive vertical speed. Note that if airspeed is decreasing as altitude is gained it is called a “zoom” not a climb.
- **Descending:** Means flight at constant airspeed and constant negative vertical speed. If airspeed is increasing while altitude is lost it is called a “dive.”
- **Turning:** Means that heading is changing.



By definition: **relative wind** is the airflow that is exactly opposite to the direction of flight, I.E. is exactly opposite to velocity. This is critical to understand. For example an aeroplane that is climbing is flying through air that passes the aeroplane in an orientation as shown in Figure 16. Similarly an aeroplane that is descending experiences airflow as in Figure 17.

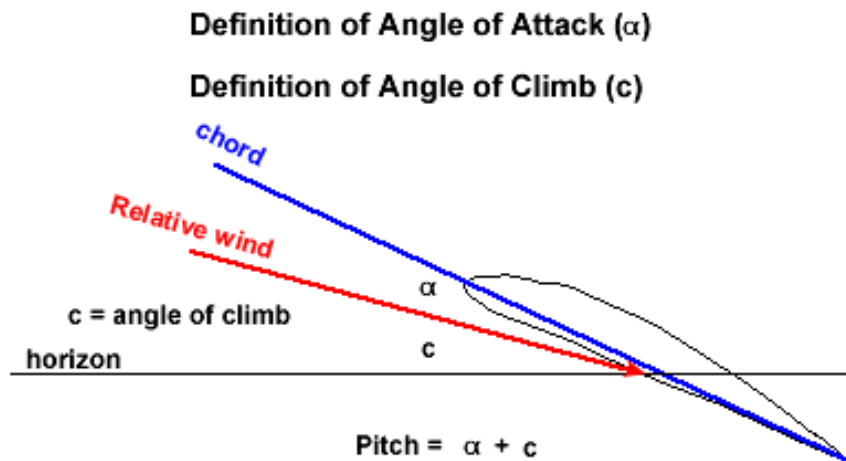


Figure 18

**Angle of attack ( $\alpha$ )** is the angle between the chord and the relative wind, expressed in degrees.  $\alpha$  is negative if the chord line is below the relative wind. Don't confuse  $\alpha$  and pitch attitude; that is a common mistake. The aeroplane in Figure 18 is climbing. It has a large pitch attitude but small  $\alpha$ . The aeroplane in Figure 19 is descending. It has a negative angle of climb but a positive  $\alpha$  and pitch attitude.

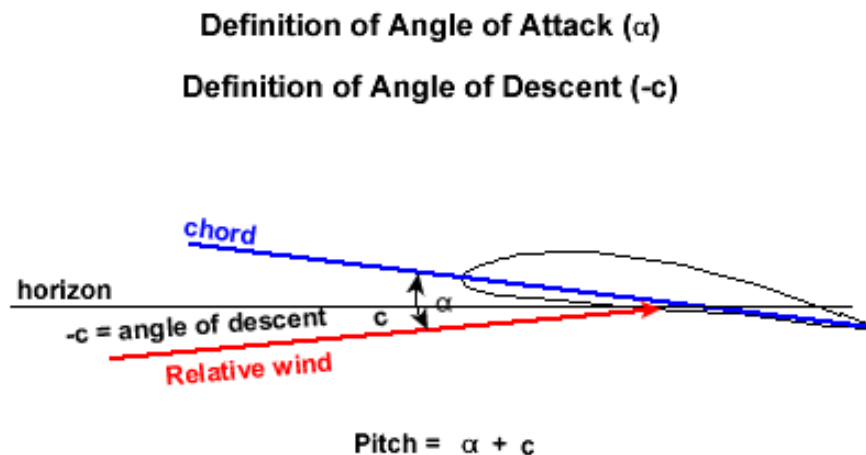


Figure 19

**Weight ( $W$ )** is the force that acts toward the center of the earth. This direction is also defined as *down*. *Up* is the opposite direction to down.  $W = mg$ . Weight acts at the center of gravity (explained below.)

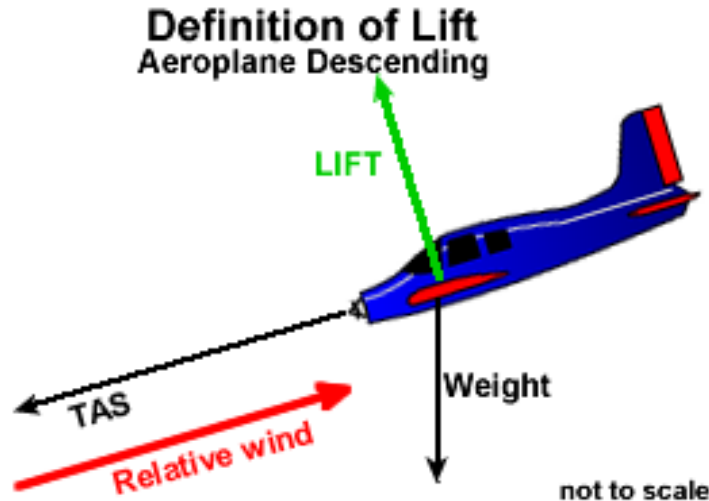


Figure 20

**Lift (L)** is defined as the force perpendicular to the direction of flight, which is the same thing as saying perpendicular to the relative wind. Be certain to fully grasp this definition because lift is NOT necessarily always “the force that opposes weight.” Failure to grasp this is often a source of great confusion. Consider Figure 20, in which an aeroplane is descending. Weight acts down, but lift acts in an inclined direction that is perpendicular to the relative wind. Many pilots get their minds stuck on the idea that lift always acts “up” and consequently have trouble grasping aerodynamics. Try to shake that idea. Lift only opposes weight in straight and level flight.

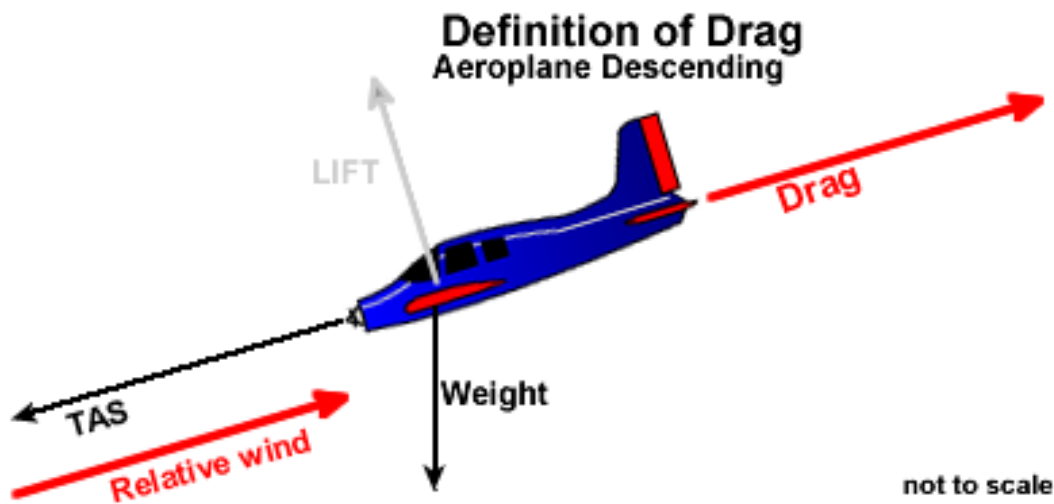


Figure 21

**Drag (D)** is the force that acts parallel to the relative wind, which is synonymous with saying opposite to the direction of flight. Consider Figure 21, where you can see that drag is not horizontal – it is parallel to the relative wind.

**Thrust (T)** is the force created by the aeroplane’s engines. Its direction depends on how the engines are installed on a particular aeroplane. In most aeroplanes thrust acts along, or



parallel to, the longitudinal axis. Sometimes the engines are installed with a very small angle to the longitudinal axis. The reason is explained on page 155.

**Center of gravity (CG)** is the point around which all moments due to gravity equal zero. CG is also the point at which weight acts. CG is such a critical concept in aerodynamics, and so poorly understood, that I am going to ask you to perform an experiment to develop a deeper understanding of the term. Please follow the instructions below:

## CG Experiment

1. Draw a picture of an aeroplane, similar to the one in Figure 22, on a piece of stiff paper or cardboard. Make the picture at least several inches long (larger is better.)
2. To more accurately represent the mass distribution of a real aeroplane tape some weight, such as a coin(s), where the engine would be on a real aeroplane. This is shown in the figure.
3. Carefully cut the aeroplane shape out using scissors. Leave as little extra paper around the outline as feasible.
4. Make several holes distributed around and near the edges of the aeroplane, as shown in the figure. These holes may be anywhere, as long as they are near the edges. Try to make the holes neatly either using a paper punch or a sharp object such as the tip of a pencil.
5. On a smooth wall drive a small finishing nail straight in so that you can hang the aeroplane on one of the holes.
6. Hang the aeroplane up and notice that it hangs at an odd angle, similar to the one in Figure 23. Try swinging the aeroplane and notice that once you let it go it swings like a pendulum for a while but it always stops in the same orientation.
7. Draw a perfectly vertical line from the nail down the aeroplane. Ideally you would use a carpenter's level or plumb bob to do this.
8. Using different holes repeat steps 6 and 7 at least twice more.
9. You should notice that all the vertical lines you draw cross at a common point. If they don't you are not being careful enough, so try again.
10. Punch another hole where the lines cross. This is the CG of your aeroplane.
11. Hang the aeroplane up by the CG.
12. Rotate the aeroplane to any pitch attitude. Notice that you can stop it at any angle (i.e. it balances in all attitudes.)

At step 12 you should have an eye-opening moment. If you didn't understand CG before you do now. The CG is the *only* place you can skewer the aeroplane and have it balance in all orientations. If you skewer it at any other point it rotates and hangs in one orientation only, and that orientation is such that the CG is below the nail.

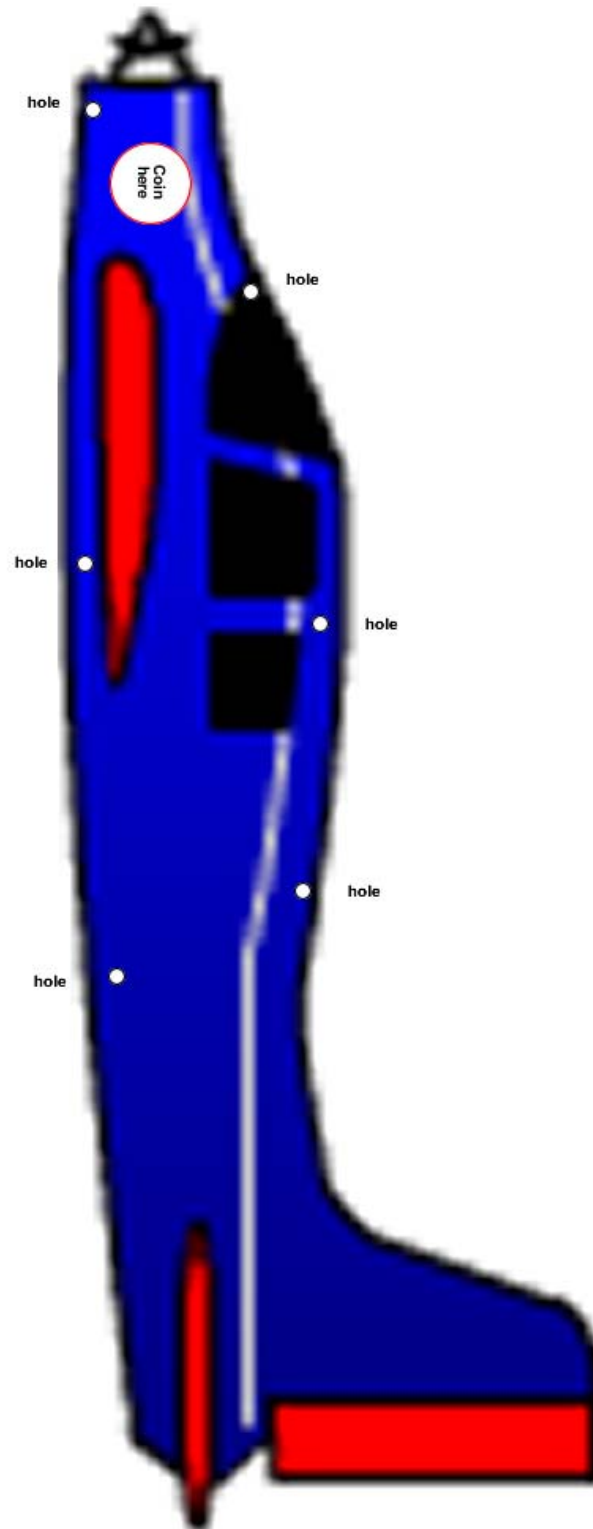


Figure 22

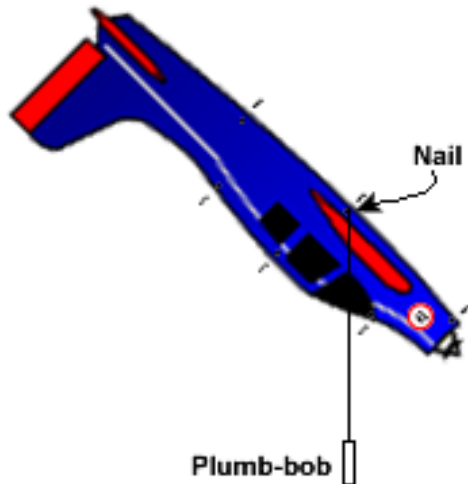


Figure 23

Everyone talks about how much **power** and **torque** the engine in their car produces, but how many of us really know what it means? In physics torque measures force times distance ( $F \times d$ ) and power measures force times velocity ( $F \times V$ .)

We will be using a unit of power called **horsepower**, which is defined as follows:

- 1.0 HP = 550 lb x ft / sec
- 1.0 HP = 33,000 lb x ft / minute
- 1.0 HP = 325.66 lb x knots
- 1.0 HP = 746 Watts

To put things into perspective, a 100-watt light bulb is equivalent to a 0.13 HP light bulb ( $100/746$ .) An electric hairdryer that consumes 1000 watts is also consuming 1.34 HP. A car engine producing 400 lb ft of torque at 5000 rpm is making 380.8 HP (calculated as  $400 \times 5000 \times 2\pi / 33,000 = 380.8 \text{ HP}^5$ .) Another engine that also makes 400 lb ft of torque but does so at 2000 rpm only makes 152.3 HP. For propeller driven aeroplanes horsepower is more important than torque, as you will see shortly.

Horsepower is measured on a dynamometer. Because the dynamometer measures torque by applying a braking action to the engine the power so measured is called **Brake Horsepower** (BHP.)

To fly an aeroplane a propeller must convert its BHP into **Thrust horsepower** (THP.) This process is never 100% effective. **Propeller efficiency** ( $\eta$ ) is defined as  $\eta = \text{THP}/\text{BHP}$ .

You may find the following two equations useful:

---

<sup>5</sup> Rpm x  $2\pi r$  gives the velocity of the object. Torque/r gives the force (r is the arm of a pulley that the engine turns and is also the arm for the weight that represents torque.)

$$\text{THP} = T \times V / 325.66 \quad [V \text{ is in knots}]$$

$$T = 325.66 \times \text{THP} / V \quad [\text{This is just the above equation solved for } T. \text{ } T \text{ is in pounds}]$$

To see the significance of the above equations answer these three questions:

1. A particular aeroplane experiences 1000 lb of drag at 100 knots. How much THP is needed for it to fly? (Tip: Thrust must equal drag in level flight.)
2. A particular aeroplane experiences 1000 lb of drag at 200 knots. How much THP is needed for it to fly?
3. A particular aeroplane experiences 1000 lb of drag at 325.66 knots. How much THP is needed for it to fly?

The answers are:

1. 307 HP
2. 614 HP
3. 1000 HP

It is important to realize that all three of the above aeroplanes require 1000 pounds of thrust to cruise, but faster aeroplanes needs more HP.

The amount of thrust a given HP engine can produce falls off with velocity. For example a 1000 HP engine produces 1000 pounds of thrust at 325.66 knots. But, how much does it produce at 10 knots (early in the takeoff roll)? To answer use the formula:  $T = 325.66 \times \text{THP} / V$ . The answer is 32,566 pounds of thrust (wow!)<sup>6</sup>

Note that when flying at 325.66 KTAS one pound of thrust equates to one THP. At lower speeds one THP produces more than one pound of thrust. Above 325.66 KTAS one THP makes less than one pound of thrust. (Since drag increases with velocity, while thrust decreases, propeller aeroplanes are quite limited in terms of how fast they can cruise. We will see that jets have an advantage in this regard.)

*It is worth noting that the equation for thrust approaches infinity as V approaches zero. It cannot actually be solved at zero velocity. The implication is that for a given amount of power thrust becomes infinite when velocity is zero, such as at the beginning of the takeoff roll. In reality that doesn't happen because the propeller becomes very inefficient at such speeds, but for propeller driven aeroplanes, since BHP output is essentially unrelated to airspeed, thrust is definitely very high at the beginning of the takeoff roll and declines as true airspeed increases. Once we complete our discussion of drag we will return to the question of the relationship between power, thrust, and velocity and analyze it further.*

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<sup>6</sup> This is likely an overestimation because we assumed the same propeller efficiency at both speeds. Even so the engine will produce a very impressive amount of thrust a low speed.

## The Sum of Forces Acting on an Aeroplane

We will consider four forces that act on an aeroplane. These are:

1. Weight
2. Lift
3. Drag
4. Thrust

Of these only weight (W) is easy to define and grasp. Weight always acts toward the center of the earth and is given by the equation  $W = mg$  (m is in slugs, and  $g = 32.2 \text{ ft/sec}^2$ ) W acts at the center of gravity.

Lift is the sum of all force acting perpendicular to the flight path, or relative wind. Lift is contributed by many parts of the aeroplane (each wing, the tail, the fuselage, perhaps other parts.) These individual forces can be summed up and represented with one vector that we perhaps should call *total lift*, but more commonly just call lift. The crucial point to note is the direction lift acts – it is **always perpendicular to the flight path**, i.e. perpendicular to the relative wind.

Drag is the sum of all forces acting opposite to the direction of flight, or parallel to the relative wind. Total drag is the sum of all the forces, contributed by each part of the aeroplane, **opposite to the direction of flight**. It too can be represented by one vector.

Thrust is the force produced by the engine(s.) It acts in the direction the engine(s) are pointed. In introductory aerodynamics courses it is customary to simplify the required analysis by assuming that thrust acts in the direction of flight, i.e. that thrust is directed opposite to drag. We will make that simplification here, but keep in mind that it is a simplification. In reality, as angle of attack changes a small component of thrust acts perpendicular to the direction of flight<sup>7</sup> (i.e. with lift.)

*Pilots should note that the definitions of lift and drag above are just that – definitions. The actual static pressure around an aeroplane creates **one** force that both supports it against gravity and impedes its forward progress. It is our convention to resolve this force into two components with the one perpendicular to the direction of flight called lift and the component parallel to the direction of flight called drag.*

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<sup>7</sup> For those who are mathematically inclined thrust equals  $\cos(\alpha)$ . Given that  $\alpha$  is seldom more than  $10^\circ$ , thrust directed along the direction of flight is normally more than 98.5% of total thrust, making the simplification acceptable.

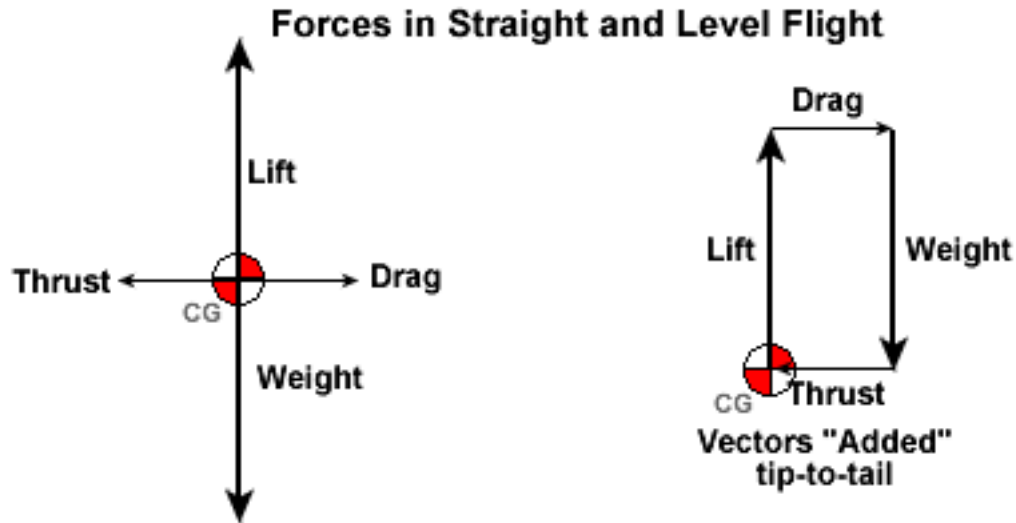


Figure 24

For our purposes we will say that in *straight and level flight*  $L = W$  and  $T = D$ , as shown in Figure 24.

The force vectors must be added to get the total force acting on an aeroplane. Vectors are added by laying them tip-to-tail (it does not matter what order.) In the cases we are interested in for now (straight and level, climbs, and descents) the total force is zero.  $W$  is the only vector that never varies.  $L$ ,  $T$ , and  $D$  change in length and orientation in accordance with their definitions, but  $L$  is always perpendicular to  $D$ . The length of  $T$  is especially variable as the pilot can change it with the throttle; armed with these thoughts follow along with the following important analysis of forces in climbs and descents.

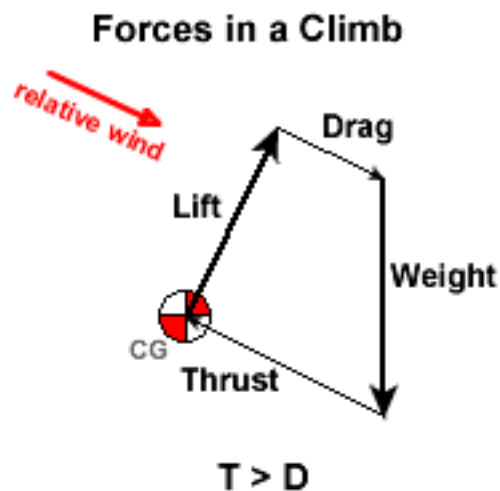


Figure 25

In a *climb*  $T$  must be greater than  $D$  ( $T > D$ ).<sup>8</sup> It is crucial to see that in every case an aeroplane that is climbing must have more thrust than drag. This situation is shown in Figure 25. It is highly recommended that you learn to draw this vector diagram and use it to confirm the simple trigonometric relationship:

**$\text{Sin}(c) = (T-D)/W$**  [ $c$  is the angle of climb. Positive means climb and negative means descending.  $C = \text{zero}$  means level flight.]

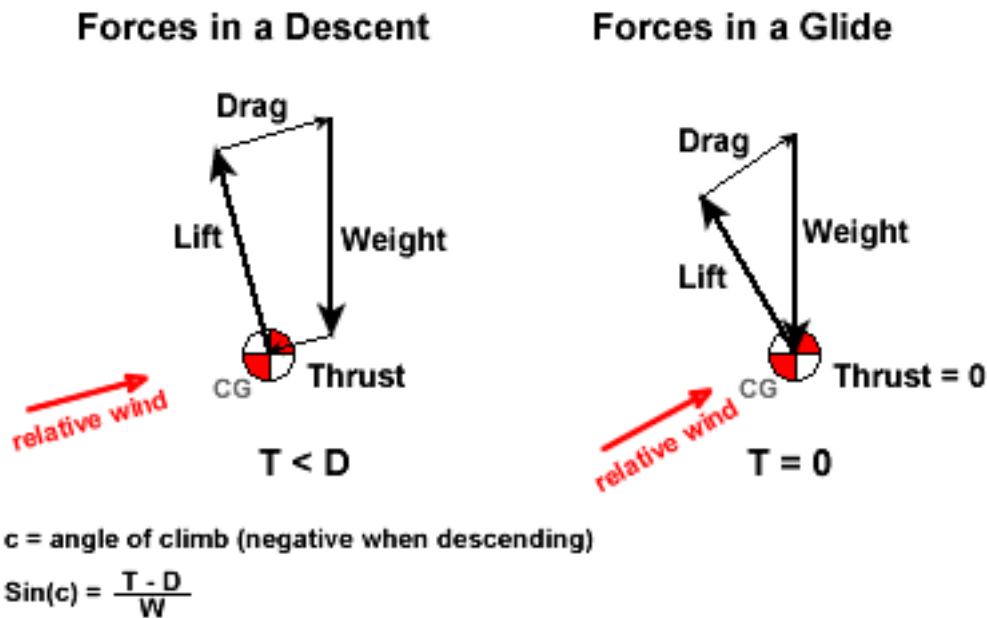


Figure 26

In a *descent*  $T$  must be less than  $D$  ( $T < D$ ). An aeroplane cannot descend unless thrust is less than drag<sup>9</sup>. Figure 26 shows the vector diagram for a descent. In the special case where  $T=0$  (i.e. the engines have all failed)  $\text{sin}(c) = -D/W$ . In other words the glide angle is determined by the ratio of drag to weight. If the pilot pushes the nose over and makes a steep glide drag is greater, and if the pilot flies at the speed for minimum drag that will result in minimum descent angle – or in other words maximum glide range. We will explore gliding more thoroughly later.

Pilots can learn a lot about how to fly an aeroplane from the equation in Figure 26. Many flight instructors use a saying something like “throttle controls altitude and elevators control airspeed.” It would actually be more precise to say that, “**thrust controls angle of climb** and elevators control airspeed”, but we are getting slightly ahead of ourselves. We will return to the question of controlling airspeed later.

<sup>8</sup> Review the definition of climb and descent given earlier. In each case the definition includes the provision of constant airspeed and vertical speed. I.E. the conditions known as zooming and diving are not considered in this text.

<sup>9</sup> Recall the previous definition of a dive. If you push the nose over without decreasing thrust the aeroplane will descend but airspeed will increase. This is called a dive, NOT a descent.

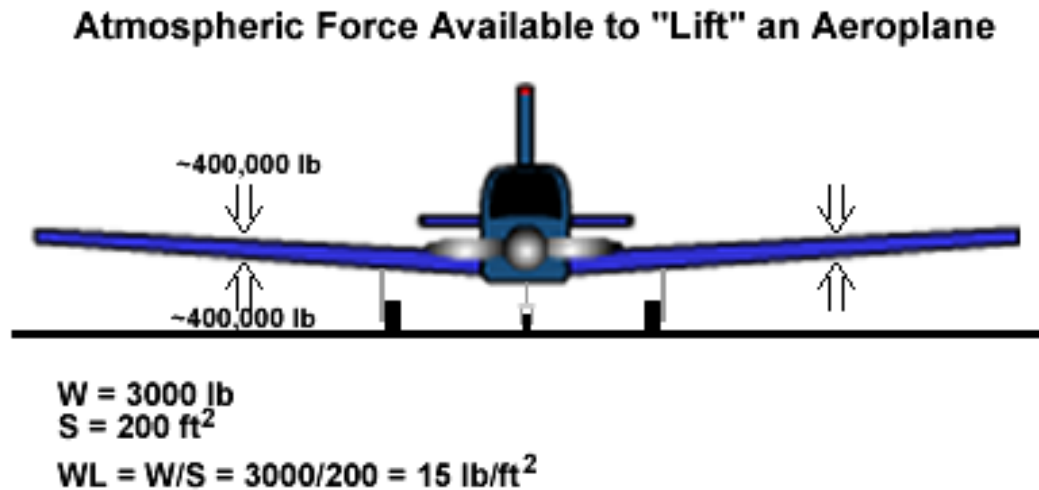


Figure 27

## The Nature of Lift

Something I would like to point out to you is that even though an aeroplane must be in motion to fly the air has enough potential energy (i.e. static pressure) to support the aeroplane with no motion required. The subject we are studying is called aerodynamics, which means "air in motion" so it is going to turn out that air-movement is necessary to flight, but it is crucial to realize that even an aeroplane parked motionless at the airport has enough air pressure under its wings to easily lift it up.

Consider the aeroplane in Figure 27, which weighs 3000 lb and has a wing area of 200ft<sup>2</sup>. This aeroplane is parked on the apron at some airport, and just sitting there a static air pressure (Ps) on the order of 2000 lb/ft<sup>2</sup> is pushing up on the bottom of the wing (the exact amount depends on the altitude of the airport, see table 1.) Therefore there is ~400,000-pound upward force that *could* lift this aeroplane into the air. Of course there is an opposing 400,000-pound force pushing down on the top of the wings that cancels out the upward push, so the aeroplane stays on the ground, held there by its weight of 3000 pounds.



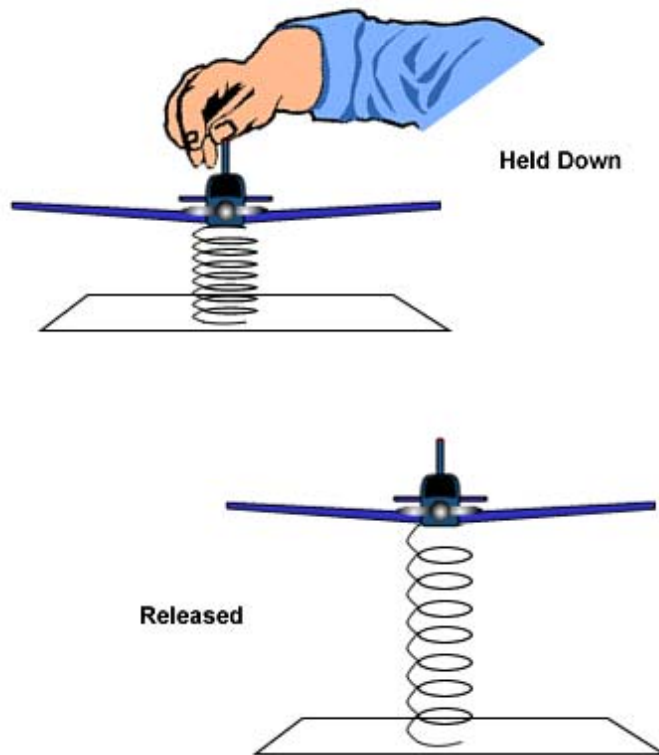


Figure 28

You may have heard that a wing produces lift by creating a low-pressure area above its top surface. This explanation is correct, but slightly misleading, as it tends to promote the false image that the aeroplane is *pulled* up by a vacuum. The real story is more like Figure 28 in which a spring-loaded toy is held down by someone's hand. As soon as the person removes their hand the toy jumps upward. Clearly the spring is responsible for lifting the toy, not the removal of the hand per se. The same is true of an aeroplane. We must realize that there is a huge force below the wing pushing upward. All that is needed is to slightly reduce the downward force, by an amount equal to the weight of the aeroplane, and then the air below the wing will easily *push* the aeroplane up into the air. To use the example numbers above, the downward force must be reduced from 400,000 to 397,000 (0.75% reduction.) A net force of 3000 will then exist, which will offset the weight and permit flight.

*For a typical light aeroplane a force of hundreds of thousands of pounds pushes both up and down on the wing. In level flight all that is needed is for the downward force to be reduced by an amount equal to the weight of the aeroplane. We now understand the true significance of the "vacuum" above the wing.*

I would like to get Bernoulli's equation involved in the explanation. Recall that we used  $P_s$  to represent the static air pressure. For an aeroplane to fly,  $P_s$  below the wing must be greater than  $P_s$  above the wing. The extent of pressure difference between the upper and lower surface of a wing is such an important factor in aerodynamics that it is given a name. It is called **wing loading** (WL.) Wing loading is one of the most important

parameters in an aeroplane's design. It directly determines the stall and cruise speeds of aeroplanes. Table 2 shows some typical wing loadings of well-known aeroplanes.

Aeroplane type	Wing loading (WL) lb/ft <sup>2</sup>
C-172	14
Beech Baron	31
KingAir	41
Lear Jet	79
A320	123
B747	146
Concorde	105

Table 2

Wing loading can be calculated by dividing weight (W) by wing area (S.) We say:

$$WL = W/S$$

The sample aeroplane from Figure 27 had a wing loading of 15, calculated as 3000/200. This is a low wing loading, typical of low-speed training aeroplanes. Faster aeroplanes such as jets have higher wing loadings, as shown in table 2.

If an aeroplane has a wing loading of 20 lb/ft<sup>2</sup> then it can be made to fly by decreasing Ps above the wing by 20 lb/ft<sup>2</sup>. For a B747 with a wing loading of 146 lb/ft<sup>2</sup> Ps above the wing must be reduced by 146 lb/ft<sup>2</sup> to keep it airborne, etc. The following two tables give wing loadings of several aeroplanes and the consequent percent change in static pressure needed to sustain flight.

Aeroplane type	Wing loading (WL) lb/ft <sup>2</sup>	Static Atmospheric Pressure at <b>sea level</b>	Percent change in Ps needed to fly
C-172	14	2116.2 lb/ft <sup>2</sup>	.7%
Beech Baron	31	“	1.47%
KingAir	41	“	1.94%
Lear Jet	79	“	3.73%
A320	123	“	5.8%
B747	146	“	6.9%
Concorde	105	“	4.96%

Aeroplane type	Wing loading (WL) lb/ft <sup>2</sup>	Static Atmospheric Pressure at <b>40,000'</b>	Percent change in Ps needed to fly
C-172	14	393.1 lb/ft <sup>2</sup>	3.6%
Beech Baron	31	“	7.9%
KingAir	41	“	10.4%
Lear Jet	79	“	20.1%
A320	123	“	31.3%

B747	146	“	37.1%
Concorde	105	“	26.7%

### ***Something to Reflect On***

The agent that actually “creates” both lift and drag is static pressure not dynamic pressure, because it is the collisions of the trillions of air molecules with the aeroplane, which is what static pressure is, that “cause” lift and drag (per rule 3 and 4.) So, why is the subject we are studying called aerodynamics and not aerostatics? Bernoulli’s equation, which tells us that when dynamic pressure changes so must static pressure, is the reason. So aerodynamics is really the study of how *changes in dynamic pressure cause changes in static pressure* and therefore lift and drag.

### ***Lift Production in Subsonic Flow***

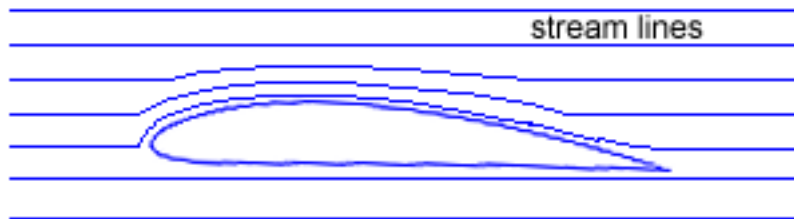
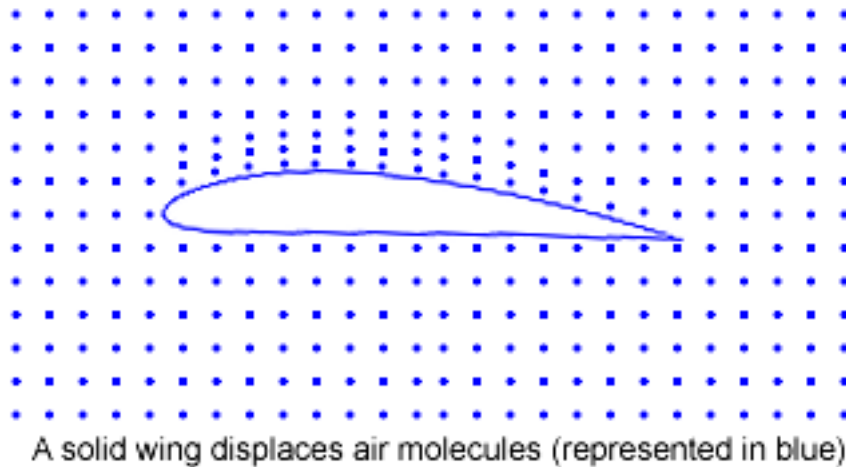
Please read the section labeled “Something to Reflect On” above.

First, realize that there is a **large** static pressure force “clamping” the aeroplane, i.e. squeezing it from above, below, front, and back. At sea level the force is more than 2000 pounds per square foot and it pushes up on the bottom of the wing and down on the top surface as well as on the leading edge, the windshield, and so on.

Second, realize that dynamic pressure, which is due to air flowing past an aeroplane in flight, represents a significant but usually rather small force compared to static pressure. Earlier we learned the equation for dynamic pressure:  $q = 1.426 \rho V^2$ . Using this equation a few representative dynamic pressures have been calculated for air flowing past some well known aeroplanes in cruise. The values are presented below:

Aeroplane	TAS (knots)	TAS Ft/sec	Altitude ft	Density Slugs/ft <sup>3</sup>	q Lb/ft <sup>2</sup>
C-172	100	169	5000	0.002048	~29
Lear jet	400	676	30,000	0.000891	~203
Concorde	1150	1942	60,000	0.000256	~482

The dynamic pressures listed in the last column represent the pressure or energy that the air flowing past these aeroplanes has. To turn it into lift however requires *concentrating* it above the wing. As calculated here it is simply an amount of energy that passes by equally below and above the aeroplane. As such it has no net effect.



The air molecules flow along stream lines as shown here.

The stream lines are forced closer together above the wing

**Figure 29**

A wing is a solid object. Common sense tells us that if we move a solid object through the air it is going to be pushed out of the way. This is shown in Figure 29. Air cannot pass through a solid wing so it is deflected, primarily along the upper surface. As the wing pushes the air molecules out of the way the stream lines, representing the paths the air molecules, are forced closer together. This is shown in the lower portion of Figure 29.

It is now time to recall the *incompressible flow continuity equation* we learned under rule 6. When the stream lines are forced closer together the inevitable result in incompressible flow is that velocity increases.

Now we invoke Bernoulli's equation:

$$P_{s_1} + \frac{1}{2} \rho V_1^2 = P_{s_2} + \frac{1}{2} \rho V_2^2 \quad [\text{Bernoulli's Equation - Metric version}]$$

$$P_{s_1} + 1.426 \rho V_1^2 = P_{s_2} + 1.426 \rho V_2^2 \quad [\text{Bernoulli's Equation - English version}]$$

The velocity of the airflow ahead of the wing is called the *free stream* velocity. Let that be  $V_1$  in Bernoulli's equation. Let  $V_2$  be the velocity above the wing, which we know must be greater than  $V_1$ . Bernoulli's equation therefore indicates that  $P_{s_2}$  must be lower than  $P_{s_1}$  (the atmospheric static pressure.)

Therefore the principles of conservation of matter and energy explain why pressure drops above a wing.

Summarizing what we now know, lift can be explained by the following sequential logic:

1. A solid wing passing through the air accelerates air over its top surface in accordance with the continuity equation.
2. In accordance with Bernoulli's equation: increased velocity causes reduced static pressure above the wing.
3. Static pressure below the wing remains unchanged and is greater than pressure above the wing\*.
4. The difference in pressure between the top and bottom of the wing *results in lift*.
5. The total lift is the net pressure pushing on the entire surface area of the wing (S.)
6. In accordance with rule 3 the wing provides an equal but opposite force to the airflow, deflecting the passing air downward.

\* Note that in the above explanation point 3 assumed that airflow along the lower surface of the wing is not changed. If  $V$  decreases along the bottom then Bernoulli's equation tells us that  $P_s$  increases. This is an alternate way of "making" lift. Most aeroplanes make some lift this way, especially when flying at high angle of attack.

## Center of Pressure

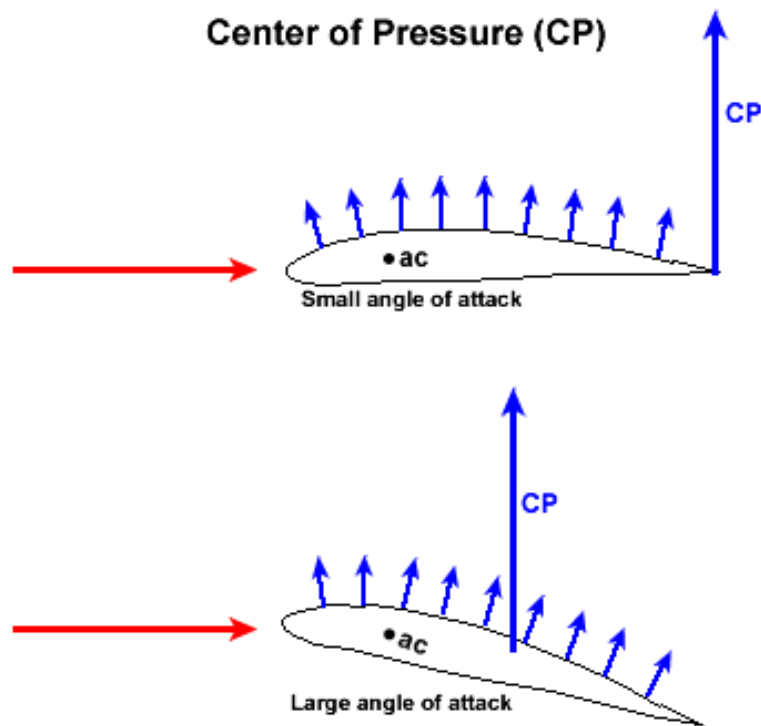


Figure 30

Each square foot of the wing produces some lift, but it should be clear that some portions of the wing produce more lift than others, because some have lower static pressure than others. Nevertheless we can sum up all the lift forces produced by the wing and represent them by a single vector. This vector acts at the *location* called the **center of pressure** (CP.)

At small angles of attack there is usually a low pressure area under the leading edge of the wing. This pressure reduces the lift contributed by the front portion of the wing. The result can be seen in Figure 30, which shows that for a cambered airfoil flying at small angle of attack the CP is well back on the wing. As angle of attack increases the CP moves forward and the forward parts of the wing become most important in generating lift.

*Special Note: On a reflexed airfoil the CP is forward on the wing, usually within 25% of the leading edge, and does not move as much when angle of attack changes.*

On a cambered wing, the imaginary point that the CP moves toward, but never reaches, is called the aerodynamic center (ac.) As angle of attack increases to near the stall CP gets closer and closer to ac but before it reaches ac the wing stalls and the CP moves aft again, which produces a nose down pitching moment. The ac is an important factor affecting longitudinal stability, covered later.

## The Lift Equation

So far we have concentrated on developing a visualization of lift, which is a pressure caused by air particles pressing on the bottom of the wing more vigorously than those pressing on the top. Now it is time to put this concept into an equation and then use it to describe several important aspects of flight.

The lift equation is:  $L = C_L \times S \times \frac{1}{2}\rho V^2$  [In metric units.]

The lift equation is:  $L = C_L \times S \times 1.426\rho V^2$  [in units described below.]

Wing area (S) is expressed in units of square feet.  $\rho$  has units of slugs per cubic foot. Lift (L) in units of pound and velocity (V) is in units of knots; this requires a velocity conversion factor<sup>10</sup>:  $1.426 = \frac{1}{2} \times (6080 / 3600)^2$ .

To understand and remember the lift equation recall Bernoulli's equation. We learned that  $P_s$  changes when dynamic pressure changes. Since lift requires a change in  $P_s$  we are not surprised that dynamic pressure appears in the lift equation.

$C_L$  is a unit-less number that expresses how much of the available dynamic pressure is converted to lift. If all the dynamic pressure is converted then  $C_L = 1.0$ , if half is converted then  $C_L = 0.5$ . It is possible to magnify the dynamic pressure so that more than 100% is converted to lift, in such a case  $C_L > 1.0$ . It turns out that  $C_L > 2.0$  is very unlikely however. We will explore this concept in detail under the heading Magnitude of Dynamic Pressure below.

$C_L \times 1.426\rho V^2$  expresses the average static pressure difference between the top and bottom of a wing; the units are pounds per square foot. This pressure value is then multiplied by wing area (S) to get the total lift. The lift equation is more logically written as:

$$L = (C_L \times 1.426\rho V^2) \times S \text{ [unit analysis: lb/ft}^2 \times \text{ft}^2 \text{ gives units of lb.]}$$

## Magnitude of Dynamic Pressure

aeroplane	TAS (knots)	Altitude ft	Density Slugs/ft <sup>3</sup>	$1.426\rho V^2$ Lb/ft <sup>2</sup>
C-172	100	5000	0.002048	
Lear jet	400	30,000	0.000891	
Concorde	1150	60,000	0.000032	

<sup>10</sup> There are 6080 ft in one nautical mile and 3600 seconds in one hour. Newton's Second Law states that  $F = ma$ . Pounds is a force unit therefore  $\text{lb} = ma$ . The definition of one pound is one slug foot per second squared. Therefore velocity in ft/sec is the required unit, and a unit conversion factor as shown must be applied.

Try this exercise, work out the value of dynamic pressure,  $1.426\rho V^2$ , for each case in the above table, and then compare your answers with the ones below. Remember that the units are pounds per square foot.

aeroplane	TAS (knots)	Altitude ft	Density Slugs/ft <sup>3</sup>	$1.426\rho V^2$ Lb/ft <sup>2</sup>
C-172	100	5000	0.002048	~29
Lear jet	400	30,000	0.000891	~203
Concorde	1150	60,000	0.000256	~483

Let's put these numbers into perspective. The numbers tell us that the kinetic energy of the "free stream" air flowing past a C-172 in cruise packs a *dynamic pressure* of 29 lb/ft<sup>2</sup>. The dynamic pressure of air flowing past a Lear Jet in cruise is about 203 lb/ft<sup>2</sup>, and the dynamic pressure of air flowing past Concorde is roughly 483 lb/ft<sup>2</sup>.

The thing to realize is that the above calculations give only the *average* or *free stream* dynamic pressure of air. Free stream means the air that is *not* affected by the aeroplane. By definition the *free stream* dynamic pressure is the same above and below the aeroplane.

As previously described, a wing actually deflects the airflow so that the dynamic pressure above the wing is greater than that below. The coefficient of lift expresses how much of the dynamic pressure is deflected. It is unit-less because it is really just a percentage that expresses what portion of the dynamic pressure is "captured" or "harnessed" by the wing to produce lift.

Lift is not like money, you don't want as much as you can get, you want just the right amount. Remember the concept of wing loading, covered previously. Table 2 specified the wing loading of a C-172 as 14 lb/ft<sup>2</sup>. Therefore an excess static pressure of 14 lb/ft<sup>2</sup> must push on the bottom compared to the top. In a cruising C-172 the dynamic pressure is 29 lb/ft<sup>2</sup> so almost half ( $14/29 = 0.48$ ) must be captured. We say the wing must operate at  $C_L = 0.48$ . You may interpret that as meaning that dynamic pressure must be 48% greater above the wing than below it. Obviously the dynamic pressure decreases if the aeroplane flies slower, so a higher percentage would need to be captured.

The Lear Jet in table 2 has a WL of 79 lb/ft<sup>2</sup>. The free stream dynamic pressure is 203 lb/ft<sup>2</sup>. A  $C_L$  of 0.39 i.e. 38% ( $79/203$ ) is required to fly the aeroplane.

The Concorde in table 2 has a wing loading of 105 lb/ft<sup>2</sup>. Calculations showed the free stream dynamic pressure is 483 lb/ft<sup>2</sup>. Consequently  $C_L$  would be 0.22 ( $105/483$ .)

Now that we fully understand the elements of the lift equation we can say that there are four key factors in the production of lift. These are:

1. Coefficient of Lift



2. Wing area
3. Air Density
4. Velocity squared

Wing area is established when the aeroplane is built so the pilot cannot change it. Air density changes with altitude, so in a sense the pilot can control it by choosing which altitude to fly at. Velocity is, within reason, under the pilot's control. But what factors determine the coefficient of lift?

### **Coefficient of Lift**

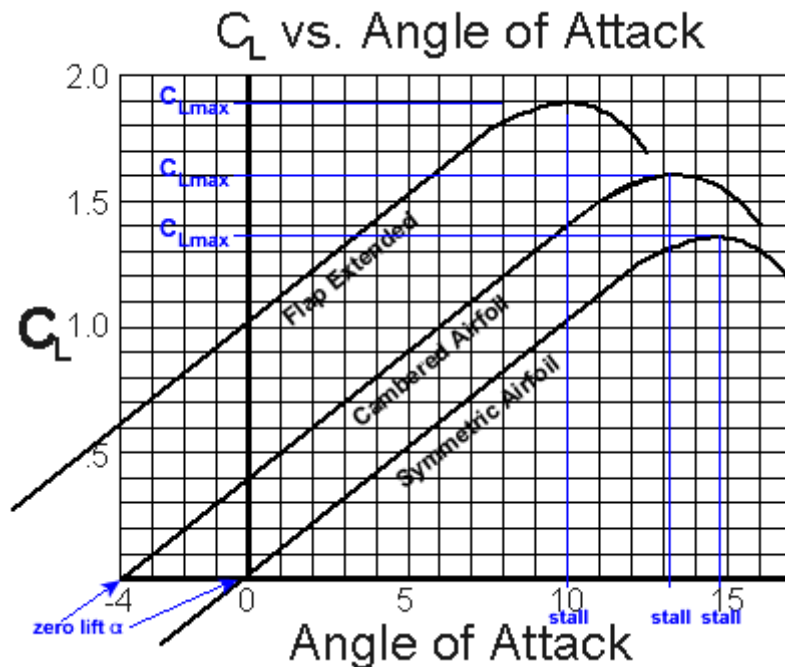


Figure 31

We know that coefficient of lift has no units. From our previous discussion about **The Nature of Lift** it is clear that  $C_L$  depends on how much the airflow velocity differs between above and below the wing. If you keep that in mind it is pretty obvious what factors affect  $C_L$ . These factors are the **thickness** and **camber** of the wing, and the **angle of attack** ( $\alpha$ .)

A thicker wing is better able to accelerate air over its top surface. Even more important is camber. Greater camber substantially increases  $C_L$ . That is why slow aeroplanes have highly cambered airfoils; while high-speed aeroplanes have airfoils with much less camber. (High speed aeroplanes have to extend flaps and leading edge devices, increasing camber, when they fly slowly.)

Thickness and camber are airfoil design parameters that a pilot cannot change in flight, with the exception of extending and retracting flaps to change camber. We will discuss the details of flaps later.

Angle of attack is the only parameter that is under the pilot's control. It is therefore the factor that pilots must use to control lift production. It is very valuable to plot a graph of  $C_L$  vs.  $\alpha$ . A typical example is shown in Figure 31.

A pilot can learn three important things from Figure 31:

1. Every wing has some angle of attack at which no lift is produced. This is called the zero lift angle of attack. It is the X-intercept on the graph.
2.  $C_L$  increases *linearly* as angle of attack increases. This is the slope of the graph.
3. There is some angle of attack at which  $C_L$  is maximized (called  $C_{L_{max}}$ .) Beyond that angle of attack  $C_L$  drops off. This angle of attack is called that *stall* angle of attack, or *critical* angle of attack. The reason for stall is explained in the section below on **Form Drag**.

Figure 31 shows three  $C_L$  vs.  $\alpha$  curves. One is for a wing with no camber. The second is for a wing with some camber, the third is for the wing with flaps extended, which is the same thing as saying a wing with even more camber.

Notice three important features of the figure:

1. The curves are parallel to each other. That means that changing angle of attack one-degree produces exactly the same change in  $C_L$  in all three cases.
2. The curves shift up and to the left as camber increases:
  - a. The maximum achievable  $C_L$ , known as  $C_{L_{max}}$  is greater with more camber.
  - b. The zero lift angle of attack becomes less (more negative) when camber is increased.
  - c. The angle of attack for any chosen  $C_L$  is less with more camber. From a pilot's point of view that means the nose must be lowered when flaps are extended. It also means that the wing stalls at a lower angle of attack with flaps extended.

2b above is particularly important for pilots to take note of. When a pilot extends flaps, in order to prevent a sudden increase in lift that causes *ballooning*, angle of attack must be decreased, which means the nose must be lowered. If the pilot does not lower the nose as flaps are extended then extra lift is generated, which results in the aeroplane being accelerated upward. This is called ballooning.

Note also that a wing stalls at a lesser angle of attack with flaps extended than clean (clean means no flaps.) On most aeroplanes flaps are fitted only to the inboard portion of a wing, so the inboard portion stalls before the outboard. This is a good safety feature

since it means that the part of the wing with ailerons does not stall and therefore roll control remains effective throughout the stall.

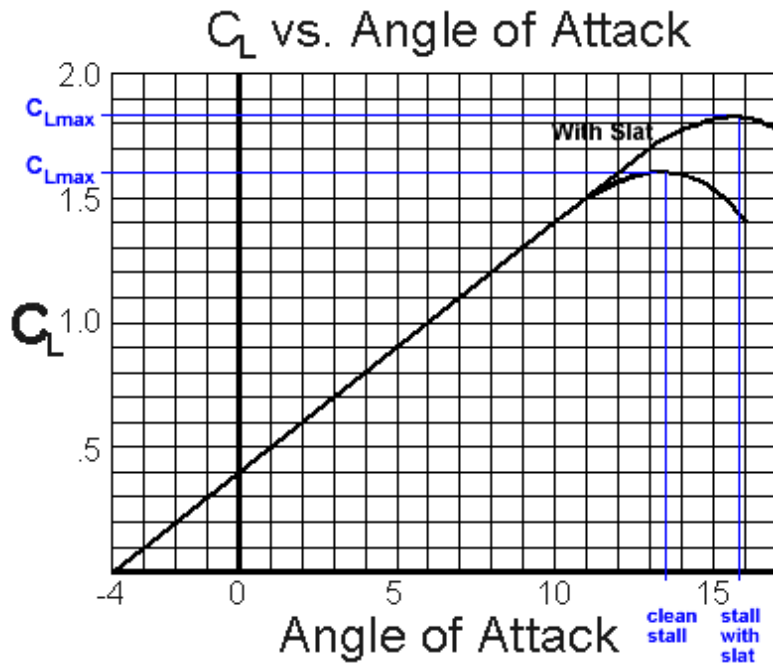


Figure 32

Figure 32 shows a  $C_L$  vs.  $\alpha$  graph for a wing with leading edge devices known as slats. We will discuss how slats work later. For now you can see that slats have no effect on  $C_L$  at small angle of attack, but they do increase  $C_{Lmax}$  and also **increase** the stalling angle of attack. An aeroplane fitted with slats is able to fly at angles of attack not possible for an aeroplane without slats.

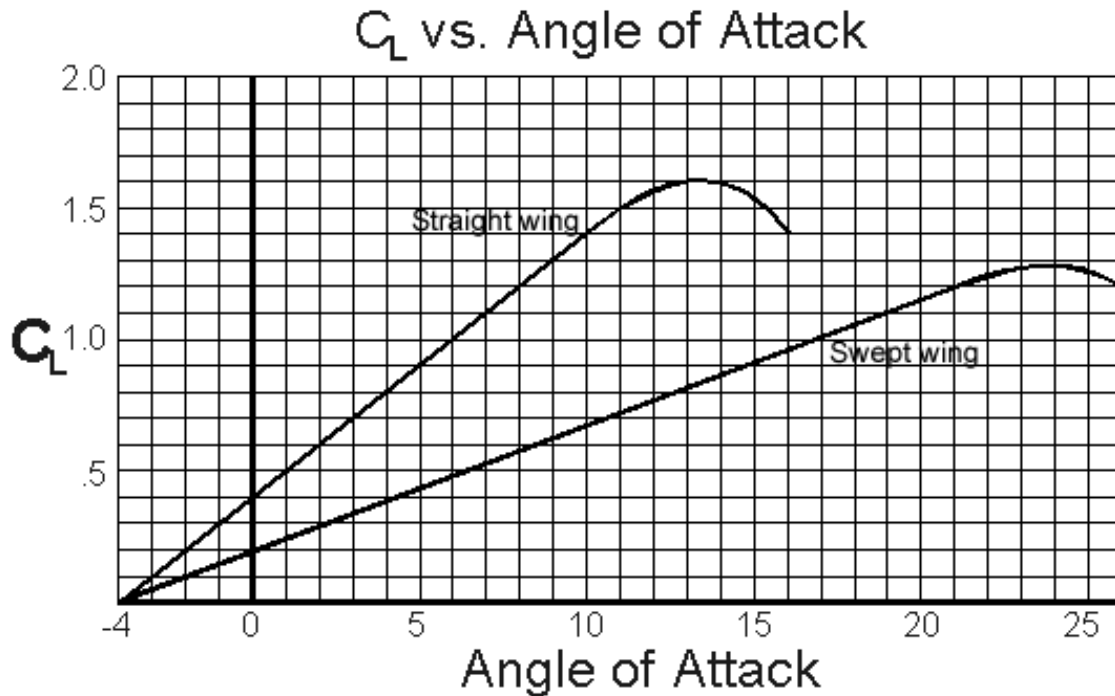


Figure 33

Figure 33 shows a  $C_L$  vs.  $\alpha$  graph for two wings. Both wings have high aspect ratio but one has no sweep (commonly called a straight wing) the other has a substantial sweep angle, like a modern jet transport. The reason for the difference will become clear later when we discuss induced drag. For now you should notice two things:

1.  $C_{L_{max}}$  is slightly lower for a swept wing than a straight wing.
2. Stall angle of attack is significantly greater for a swept wing. Consequently a swept-wing aeroplane can fly at angles of attack not possible for straight-wing aeroplanes.

If you have ever gone to the airport to watch aeroplanes landing you can now explain several observations you may have made:

- Most large jets have swept wings and leading edge slats. Consequently they can and do fly at very large angles of attack when approaching to land. We see this as a nose up attitude on approach.
- Most turbo-props (e.g. Dash-8) have straight wings and do not have leading edge slats. These aeroplanes do however have large flaps that substantially reduce the stalling angle of attack. These aeroplanes approach for landing at much smaller angle of attack, which means they fly much more nose down on approach than jet transports.

- Many business jets and regional jets have swept wings and flaps but no leading edge slats. These aeroplanes approach at attitudes between the above two examples.

Hopefully the preceding has provided some insight into how aerodynamics relates to what we do as pilots.

### ***Coefficient of Lift for Un-accelerated Flight***

One of things every pilot needs to grasp is that there is **one, and only one**, angle of attack that **must** be used for each flight speed in **un-accelerated flight**. When an aeroplane flies slowly more angle of attack is needed, when it flies faster less is needed.

The  $C_L$  vs.  $\alpha$  graph relates each angle of attack to one unique coefficient of lift. We could write another equation to express this, but let's just plan to use the graph to establish the correspondence.

We can rearrange the lift equation to solve for  $C_L$ :

$$C_L = L / (S \times 1.426\rho V^2)$$

In level flight  $L = W$ . Earlier we saw that in moderate climbs and descents on constant heading  $L$  is just slightly less than  $W$ . In this book we are going to limit our discussion to the case of constant airspeed. Consequently we can develop a very useful equation for straight level, climbs and descents i.e. un-accelerated flight:

$$C_L = W / (S \times 1.426\rho V^2) \quad [\text{for straight flight, i.e. when } L = W]$$

A pilot flying an aeroplane is constantly solving this equation, even though s/he doesn't realize it. In order to maintain level flight the pilot uses the elevator to change angle of attack to get exactly the correct  $C_L$  for the velocity. Whenever  $V$  changes the pilot adjusts angle of attack to establish the required  $C_L$ .

Try using the equation to calculate some representative  $C_L$  values for your aeroplane at various speeds ranging from just above stall to high-speed cruise. Lookup the required angle of attack in the graph, or even better in a reference book such as *Theory of Airfoil Design* and reflect on the pitch attitude you must maintain at each of these airspeeds. You might find the following equation helpful:

$$\text{Pitch attitude} = \text{Angle of Attack} + \text{angle of climb} - \text{angle of incidence}$$

*Angle of incidence* is the angle that the chord of the wing is attached relative to the longitudinal axis of the aeroplane. It is set by the aircraft designer so that the fuselage will be level (i.e. pitch = 0) at the normal cruise angle of attack. In this text I most ignore angle of incidence treating it as zero.

## **Control of Airspeed – Preliminary Thoughts**

We know that there is a unique angle of attack for each airspeed in un-accelerated flight. An aeroplane whether flying level, climbing, or descending, at a given airspeed, is always at the same angle of attack.

Hopefully you are convinced that a pilot must change angle of attack when airspeed changes, but does that mean that airspeed will change if the pilot changes angle of attack? That is indeed what happens, but we will have to wait until we have analyzed drag before we can conclusively prove it.

## ***Stall Speed Equation***

A complete description of why a wing stalls will have to wait until we discuss drag in the next section. For now we can develop an equation for stall speed.

In the CARs stall speed ( $V_s$ ) is defined as the lowest speed at which an aeroplane is controllable in straight and level flight. As we saw in the equation  $C_L = W / (S \times 1.426\rho V^2)$  above that reduced speed requires increased coefficient of lift.

To stall an aeroplane the wing must be “forced” to a large angle of attack. I say “forced” because if an aeroplane is stable it resists being stalled (we will discuss what makes an aeroplane stable later.) To keep any wing at an angle of attack more than its zero-lift angle of attack a moment is needed. Without this moment the wing would pitch to the zero lift angle of attack and the airplane would plunge from the sky like a lawn dart. Most aeroplanes have a stabilizer equipped with elevators behind the main wing that holds the tail down maintaining the wing at useable angle of attack. The more the tail pushes down the greater the angle of attack the wing flies at. Some aeroplanes, known as Canards, have elevators ahead of the main wing. The Canard lifts the nose of the aeroplane to increase the wing’s angle of attack.

To increase a wing’s angle of attack the stabilizer operates at ever more negative angle of attack (or the Canard operates at ever more positive angle of attack.) It is possible to design an aeroplane such that the stabilizer (or Canard) stalls before the main wing. If that is done then the maximum coefficient of lift achievable ( $C_{L_{\text{max-achievable}}}$ ) is less than  $C_{L_{\text{max}}}$ .

Most conventional aeroplanes achieve  $C_{L_{\text{max}}}$ , which is to say they can be “fully” stalled. In developing the stall speed equation we will assume that that flight at the stall speed means that the wing is at  $C_{L_{\text{max}}}$ .

In straight and level flight  $L = W$ . Consequently,  $V_s$  can be found by rearranging the lift equation to give:

$$V_s = \sqrt{\frac{W}{S \times C_{L_{\max}} \times 1.426 \rho}}$$

Recall the W/S is called wing loading (WL) so the equation can also be written:

$$V_s = \sqrt{\frac{WL}{C_{L_{\max}} \times 1.426 \rho}}$$

The above equation will give the un-accelerated **true** stall speed, in knots, of any aeroplane. To get the calibrated stall speed set  $\rho = 0.002377$  (see table 1.)

Notice that four factors establish the stall speed of every aeroplane. The first two factors combine as wing loading ( $WL = W/S$ ):

1. Weight (W)
2. Wing Area (S)
3.  $C_{L_{\max}}$
4. Air density ( $\rho$ )

Notice that  $V_s$  is proportional to the square root of the factors. Consequently a change in aeroplane design such as doubling the wing loading will affect stall speed by the square root of 2 (1.41.)

Wing loading is the dominant design parameter that determines stall speed. Aeroplanes with higher wing loading always have high stall speeds (refer back to table 2.) Cutting wing loading in half reduces stall speed to about 70% (1/1.41) of the previous value.

If the Pilot Operating Handbook for a particular aeroplane does not divulge the stall speed at weights other than maximum they can be calculated using the equation:

$$V_s = V_{s_{\text{published}}} \times \sqrt{W / W_{\text{published}}}$$

$C_{L_{\max}}$  is the other *design* parameter that determines stall speed. The value of  $C_{L_{\max}}$ , as we have seen, depends on camber and the use of high lift devices such as slats and flaps. Aircraft designers must balance the cost and weight of such devices against the desire to have a low stall speed. Typical  $C_{L_{\max}}$  values for a wing with no flaps is 1.4, with a simple flap it might be 1.6 and with expensive slats and a complex flap system it might reach 2.0. For small aeroplanes low wing loading is the most practical way to obtain a low stall speed.

Remember that air density decreases with altitude, so pilots must realize that true stall speed increases with altitude. Equivalent stall speed (EAS) does not change with altitude. Ignoring compression error the calibrated (CAS) and indicated (IAS) stall speed can also

be considered unchanged by altitude<sup>11</sup>. But, always remember that the true stall speed does increase with altitude, which substantially increases the length of runway needed at high altitude.

Remember that the  $V_s$  equation is only for un-accelerated flight. *Lift equals weight* only in straight and level flight, climbs and descents. Lift must be more than weight in a turn. In the next section we define load factor in a turn and in the following section we define stall speed in a turn.

### Load Factor

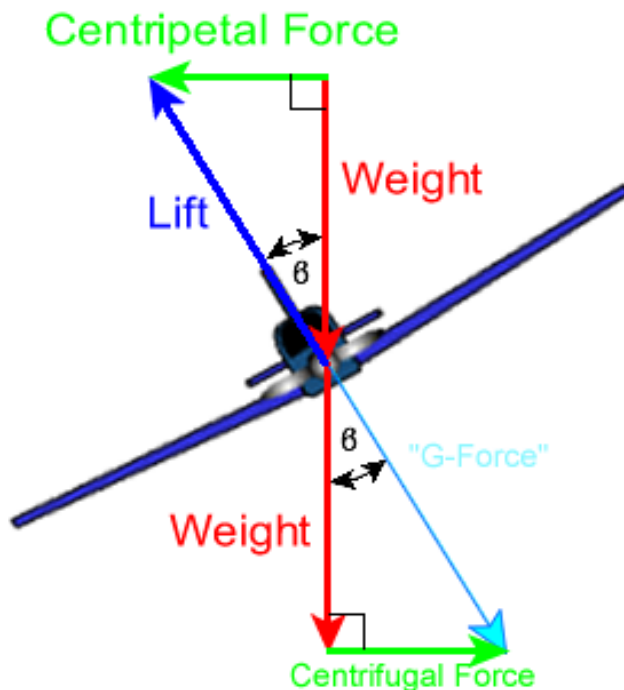


Figure 34

In straight and level flight we want lift to be precisely equal to weight. But in a turn we need more lift. Figure 34 shows an aeroplane in a coordinated<sup>12</sup> turn with bank ( $\beta$ .) The amount of lift ( $L$ ) required is given by the function  $L = W \cos(\beta)$  where  $\beta$  is the angle of bank.

It is common practice to define a **load factor** (LF) as  $LF = L/W$ . In straight and level flight we fly at a load factor of 1.0, i.e. lift equal to weight. In a turn lift is more than weight so the load factor is more than one. The simple equation  $LF = 1/\cos(\beta)$  should be memorized and a few sample values committed to memory.

<sup>11</sup> Strictly speaking it is the equivalent stall speed that is constant with altitude. Due to compression error there is a slight increase in indicated stall speed with altitude. For light aeroplanes this is less than one knot at the service ceiling, but can be several knots for a large jet. Compression error and equivalent speeds are covered in the navigation section of the text.

<sup>12</sup> A coordinated turn means that the aeroplane is not slipping in the turn.



Load factor has no units, because it is the ratio of two forces. Nevertheless pilots commonly invent a unit called “g” for load factor. Therefore we fly most of the time at 1g, in turns we experience more than 1g. For example, in a 60° bank turn we experience 2g.

### ***Stall Speed in a Turn***

The stall speed equation developed previously assumed  $L=W$ , which is correct for un-accelerated flight. In a turn lift is given by  $L = W / \cos (\beta)$ , as explained under **Load Factor**. We can therefore create an equation that relates the published stall speed to the stall speed at a given angle of bank  $V_{sb}$ .

$$V_{sb} = V_s / \sqrt{\cos (\beta)}$$

An alternate form of this equation can be derived by remembering that  $LF = 1/\cos (\beta)$ . Solving for  $\cos (\beta)$  and substituting we get:

$$V_{sb} = V_s \sqrt{LF}$$

It is very worthwhile to prepare a table showing the load factor and stall-speed-factor at a few angles of bank. The intent is to put the effect of bank into perspective for the pilot.

Angle of bank (b) degrees	Load Factor LF	Stall speed factor $\sqrt{LF}$
0	1	1
10	1.01	1.007
30	1.15	1.07
45	1.41	1.19
60	2.0	1.41

Taking a 45° bank as an example, load factor in the turn is 1.41g but the stall speed only increases by 19%.

### ***Design Load Limit and Maneuvering Speed***

Every aeroplane is designed to withstand a certain maximum positive and negative load factor. Load factor was defined previously as  $LF = L/W$ . The limits are specified in the CARs. For normal, utility and commuter aeroplanes CAR 523 applies. For transport category aeroplanes see CAR 525.

CAR 523.337 specifies the maximum load limit (n) for normal, utility, and commuter aeroplanes. It reads as follows:

523.337 *Limit Manoeuvring Load Factors*

(a) The positive limit manoeuvring load factor  $n$  may not be less than:

(1)  $2.1 + 24,000/(W + 10,000)$  for normal and commuter category aeroplanes, where  $W$  = design maximum takeoff weight, except that  $n$  need not be more than 3.8

(2) 4.4 for utility category aeroplanes; or

(3) 6.0 for aerobatic category aeroplanes.

(b) The negative limit manoeuvring load factor may not be less than:

(1) 0.4 times the positive load factor for the normal, utility, and commuter categories; or

(2) 0.5 times the positive load factor for the aerobatic category.

(c) Maneuvering load factors lower than those specified in this section may be used if the aeroplane has design features that make it impossible to exceed these values in flight.

Using the equation from the CARs, a light aeroplane weighing less than 4118 lb is required by regulation to have a maximum positive load factor of +3.8 and a maximum negative load limit of -1.52. Any aeroplane heavier than 4118 pound may have a lower maximum load limit. For example a commuter category aeroplane weighing 30,000 lb would only require load factors of +2.7 and -1.08.

The actual maximum load factor for the aeroplane you fly will be specified in the aircraft POH.

CAR 523.335c defines maneuvering speed  $V_a$  based on the design load factor as follows.

(1)  **$V_a$  may not be less than  $V_s \sqrt{n}$**

Note that  $V_a$  equals stall speed multiplied by the square root of the design load factor. For example if  $n=3.8$  then  $V_a = V_s \times \sqrt{3.8}$ , which is approximately twice the stall speed. The same factors that affect stall speed (previously discussed) affect  $V_a$ .

The equation above shows how Transport Canada defines maneuvering speed, but what does it mean to a pilot?

Maneuvering speed is really just the stall speed at the design load factor. In other words it is the speed at which the aeroplane will stall if “pulling”  $n$  gs (say 3.8g for a typical light aeroplane.) Mathematically you can see that if you compare the maneuvering speed

equation to the equation for stall speed in a turn<sup>13</sup>. This does not mean  $V_a$  only applies in steep turns.  $V_a$  is the speed at which the aeroplane stalls any time  $L = nW$ . This could occur in a steep turn (e.g. 75° of bank for 3.8g) or in another accelerated maneuver, such as a loop. One thing we can say for sure. If the aeroplane stalls at the maneuvering speed the aeroplane will be following a curved flight path and there will be a high g-force.

It is important to note that when an aeroplane flies slower than  $V_a$  it is impossible to exceed the design load limit, because the aeroplane will stall first. Consequently, as you may have read elsewhere,  $V_a$  is the speed below which a pilot can use full elevator-control deflection without risk of over stressing the aeroplane.

Many pilots are confused about the effect of weight on  $V_a$ . As the aeroplane gets lighter you can see from the equations that  $V_a$  gets lower (as does  $V_s$ .) In other words the lighter the aeroplane the lower the maneuvering speed. To many pilots this seems backward because there would obviously be less stress on the wings at lower weights. That is correct; however you are encouraged to read CAR 523.347 through 523.397. In a nutshell, even though stress on the wings may decrease as weight decreases stress on other parts of the aeroplane such as the fin, tail, engine mounts, seats, etc. does not. Even if you don't fully understand the technical specifications you must accept that you can only apply full control deflections at speeds below  $V_a$ , as specified in the CARs. Therefore pilots must not exceed  $V_a$  when making full control inputs, even at reduced weight.

One final crucial point about  $V_a$  must be made. CAR 523.345 specifies that the maximum load limit with flaps extended need only be 2.0g. Consequently  $V_a$  only applies with flaps retracted; once flaps are extended the pilot must take care to avoid high-g maneuvers. It is generally wise to avoid use of flaps in severe turbulence for this reason.

### ***Vertical Gust Loads***

A vertical gust is commonly called an updraft or downdraft by pilots. It is a column of air that is rising or descending. When a wing encounters an upward-gust, angle of attack increases and therefore lift increases, possibly resulting in violent "g-forces."

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<sup>13</sup>  $V_{sb} = V_s \text{ oLF}$  therefore  $V_s = V_{sb} \text{ oLF}$

### Up-gust Effect on Angle of Attack

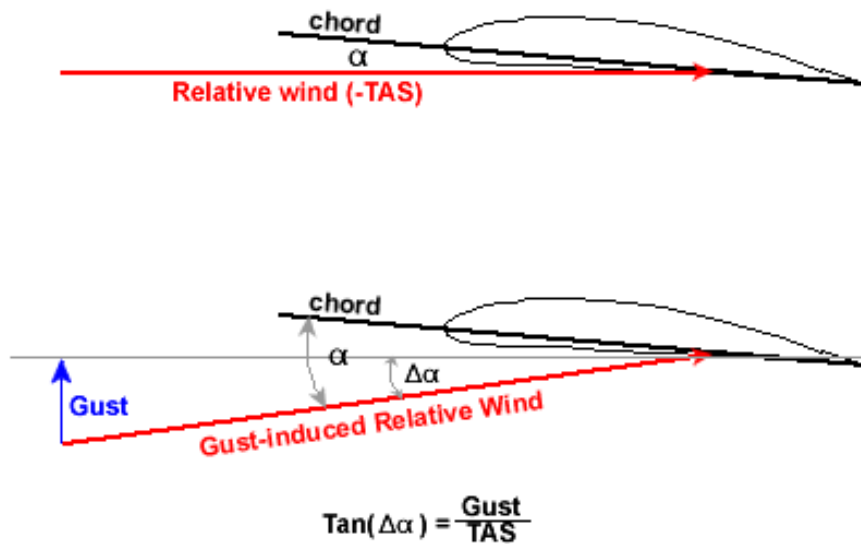
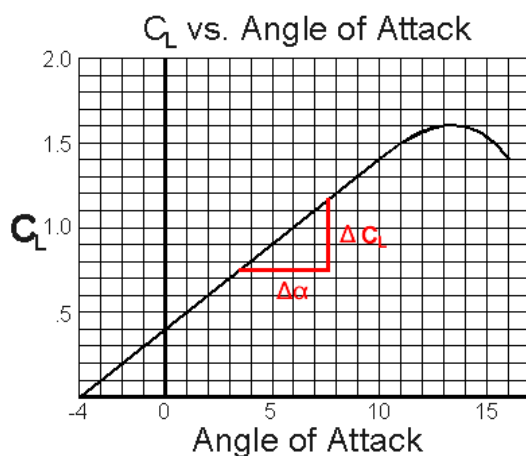


Figure 35

Figure 35 shows that the change in angle of attack ( $\Delta\alpha$ ) is governed by the relationship:

$$\text{Tan}(\Delta\alpha) = \text{gust/TAS}$$

This means that greater gust results in more  $\Delta\alpha$ , on the other hand the faster the aeroplane (high TAS) the less  $\Delta\alpha$ . Of course any  $\Delta\alpha$  results in  $\Delta C_L$ .



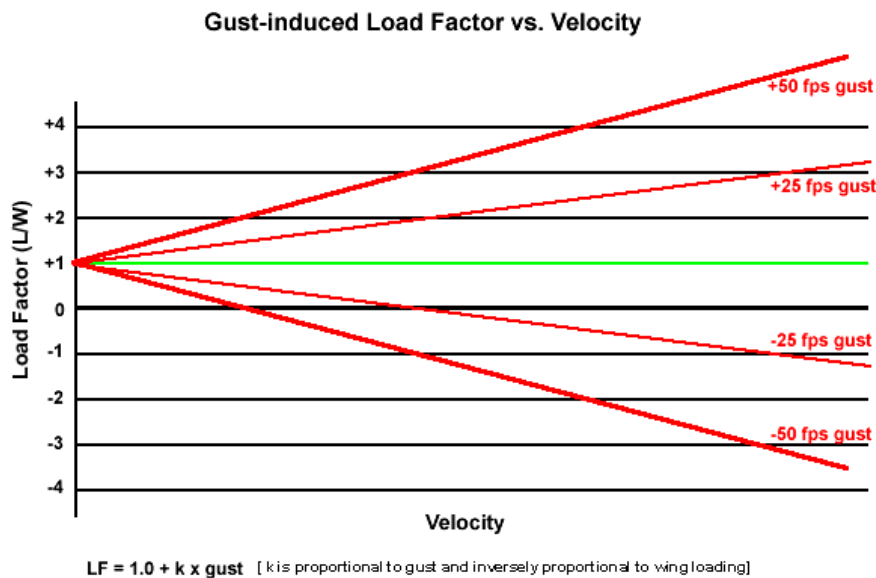
Because the lift equation shows that Lift is proportional to  $V^2$  we might have predicted that the “impact” of a gust would also be proportional to the  $V^2$ , but because  $\Delta\alpha$  is inversely proportional to  $V$  the actual effect of a gust is proportional to  $V$ . Figure 36 shows a plot of gust-load-factor vs. velocity in un-accelerated flight (i.e. starting  $LF = 1$ ) for 25 and 50 ft/sec up gusts. The equation of these lines is of the form:

$$LF_{\text{gust}} = 1.0 + k (V \times \text{gust})$$

The number 1.0 indicates that if there is no gust then the load factor is 1.0. The factor (V x gust) tells that gust-load depends on both TAS and the strength of the gust. The interesting question is what determines the value of k (slope of the line)? There are two factors.

1. Wing loading
2. Wing design

High wing loading means a lower value of k. In other words an aeroplane with high wing-loading “feels” turbulence less.



**Figure 36**

Wing design has a smaller effect because k, which depends on the slope of the  $C_L$  vs.  $\alpha$  curve. Swept-wing aeroplanes are affected slightly less by turbulence because  $C_L$  changes less for a given  $\Delta\alpha$ .

Don't get bogged down in the math of all the points above. What a pilot usually needs to know is how to evaluate another pilot's report of turbulence. First it is important to read the MET section of your AIM and commit yourself to report turbulence according to the approved protocols. Even if all pilots do that it is still important to realize that moderate turbulence for one aeroplane could be severe for another and light for still another. For example, if a Learjet reports moderate turbulence what would you expect in your aeroplane?

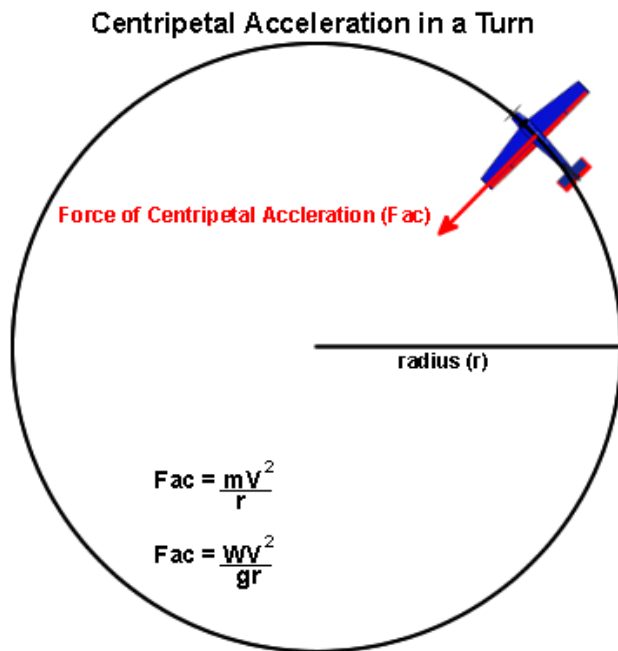
If your aeroplane flies half the speed of the Learjet, then all things being equal the turbulence will be only half as bad for you. On the other hand if your aeroplane's wing

loading is  $\frac{1}{4}$  that of the Learjet that would make the turbulence four times worse for you; if both these apply then it will be about twice as bad for you as the Learjet. If your aeroplane has straight wings (the Lear has swept wings) that will make things even worse for you.

On the other hand, if you are flying a large jet with wing loading twice that of the Lear but comparable speed and wing sweep you should expect about half the turbulence severity.

The purpose of the above analysis is to give you some idea how to put turbulence reports into perspective. Don't attempt to be too mathematical, but do remember what factors determine the severity of turbulence.

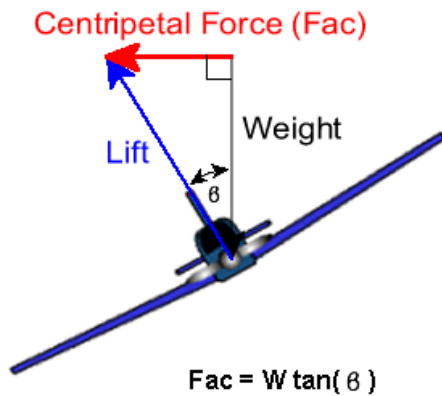
## ***Radius of Turn***



**Figure 37**

Consider Figure 37, which shows a top view of an aeroplane flying a circular flight path. The aeroplane is experiencing a force, called centripetal force, pulling it toward the center of the turn. Newton calculated that the centripetal force  $F_{ac} = mV^2/r$  ( $r$  is radius of the turn.) Since we know the relationship between mass and weight we can rewrite Newton's equation as:

$$F_{ac} = (W/g) V^2/r$$



**Figure 38**

Figure 38 shows a view from behind an aeroplane in a turn, where we can see  $F_{ac}$ , which is really just the horizontal component of lift. Keep in mind that this is the same force as in Figure 37, which viewed it from above. From the diagram we can see that:

$$F_{ac} = W \tan(\beta)$$

Set the two equations equal to each other and solve for  $r$ . The equation obtained needs a unit correction factor. Once that is introduced we get:

$$r = \frac{\left(\frac{6080}{3600}\right)^2 V^2}{g \tan(\beta)} \quad [r \text{ is in units of feet.}]$$

The desired units for  $r$  are ft (as opposed to nautical miles) The common units of  $g$  are  $\text{ft/sec}^2$ , therefore it is necessary to include a conversion factor.. The required factor is  $(6080/3600)^2$ . With this factor included we can use knots as the velocity unit and set  $g$  equal to  $32.2 \text{ ft/sec}^2$ , which gives the equation:

$$r = \frac{V^2}{11.289 \tan(\beta)}$$

Three important things can be learned from this equation:

1. Weight cancelled out of the equation. So **weight has no bearing on radius of turn**. A large aeroplane turns at exactly the same radius as a small aeroplane.
2. **Radius of turn is proportional to velocity squared**. Doubling velocity quadruples turn radius.
3. Radius of turn is inversely proportional to  $\tan(\beta)$ . Steeper turns have smaller radii.

Figure 39 shows a plot of radius-of-turn vs. velocity at various bank angles. It shows quite clearly that increasing bank has a dramatic effect on reducing radius of turn. Of course we already know that LF and Vs increase in steep turns so there must be some limit. An interesting question that pilots face, especially when flying in confined areas such as mountain valleys, is what speed and angle of bank is best to use to achieve minimum possible radius. What do you think?

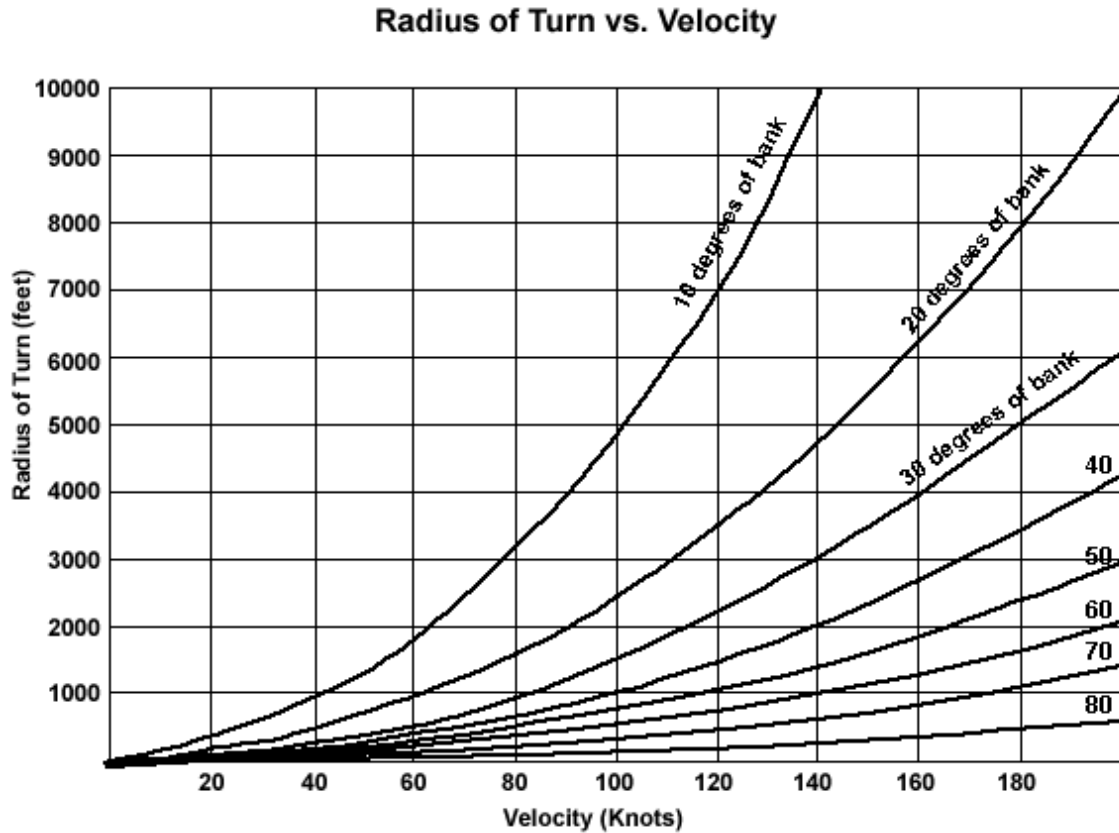


Figure 39

Figure 40 repeats the graph of the previous figure but with a red line representing stall speed. The actual stall speed of the aeroplane you fly will be different, but the way bank affects it is the same. Notice that stall speed increases only slightly up to 30° of bank, and then escalates rapidly. To keep from stalling, an aeroplane must always remain above and to the right of the red line.



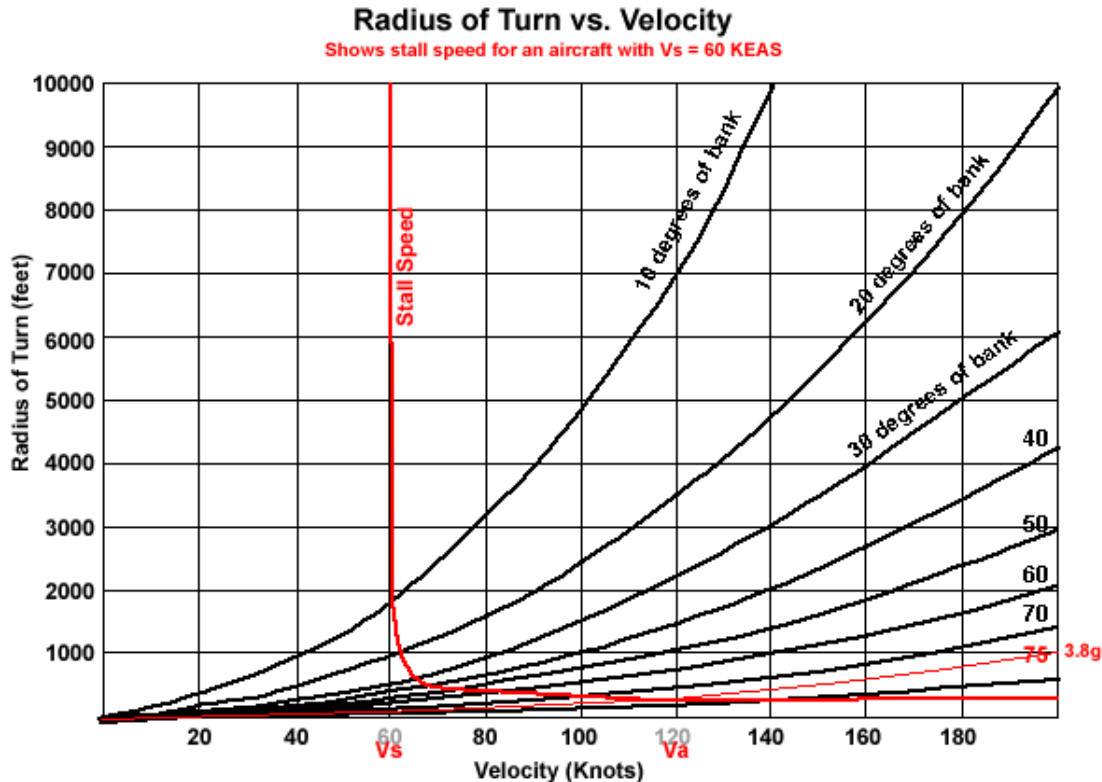


Figure 40

At  $\sim 75^\circ$  bank another red line is drawn in labeled 3.8g (remember that  $LF = 1/\cos(\beta)$ .) Bank beyond this value will overstress most light aeroplanes. Theoretically, minimum radius of turn occurs where the two red lines cross. What speed is that?

The speed is  $V_a$ . In fact  $V_a$  is called the “maneuvering” speed because it represents the speed for tightest turn (it is a military term relating to dog-fighting tactics.)

For a civilian pilot considering what to do when needing to turn in a small area it is wise to see that only a few percentage points of turn radius are given up if the turn is limited to  $45^\circ$  of bank. Since this drops the g-force from 3.8, which will blackout most pilots, to a more comfortable 1.41g it is highly recommended that pilots limit steep turns to  $45^\circ$  bank. A reasonable safety margin above the stall speed must also be maintained. Since stall speed at  $45^\circ$  bank is  $1.19 \times V_s$  then something like  $1.3 \times V_s$  would provide a reasonable safety margin.

One final point about turning with a small radius should be made. Since pilots should limit the turn to  $45^\circ$  bank then flaps can be used. As previously pointed out, when flaps are extended most aeroplanes are limited to 2g, so a 3.8g turn would be prohibited. But there is no problem using flaps at  $45^\circ$  bank, and the lower stall speed ( $V_{so}$ ) may produce a tighter turn than a  $75^\circ$  bank turn at  $V_a$ .

## Rate of Turn

Rate of turn refers to how long it takes to turn.

Most pilots are familiar a “rate one turn,” which is a turn that requires two minutes to turn 360°. You may have heard the rule of thumb that to make a rate one turn your bank angle must be ten percent of your TAS plus seven. Where does this “rule” come from and how accurate is it?

Rate of turn is expressed in units of degrees per second, which refers to degrees of heading change (HC) per second. We will use the expression ttt to represent time to turn. Therefore rate of turn (ROT) is:

$$\text{ROT} = \text{HC} / \text{ttt}$$

### Time To Turn (ttt) and Bank for Rate One Turn

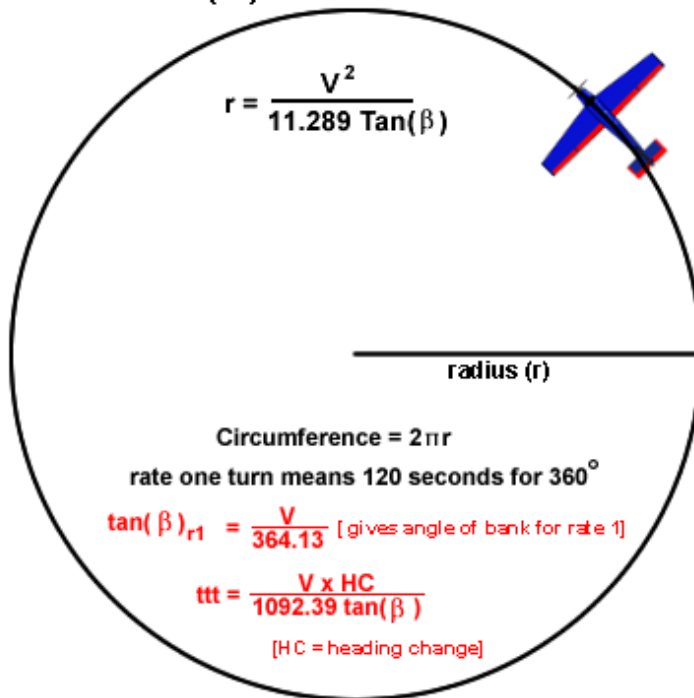


Figure 41

## Rate of Turn (ROT)

Consider Figure 41, which reminds us that the circumference of a circle equals  $2\pi r$  and repeats the radius of turn equation developed earlier. The equations shown in red indicate the angle of bank for a rate on turn and the time to turn at any velocity and angle of bank combination. The later equation is:

$$t_{tt} = \frac{V \times HC}{1092.39 \tan(\beta)}$$

Solving the equation for rate of turn (ROT) as defined above we get:

$$ROT = (HC / t_{tt}) = 1092.39 \times \tan(B) / V$$

Try out the above equation with a few sample calculations. For example cruising in a C-172 at 100 knots and making a 15° bank, what is the rate of turn? You will get 2.9, which is just a smidge less than a rate-one turn (3° / sec.) From your experience this should sound about right. Try again for a 17° bank turn.

The equation is not especially useful in flight but it is instructive as it shows that rate of turn is *inversely proportional to velocity*, NOT velocity squared as we might have supposed. So an aeroplane traveling twice as fast takes twice as long to turn, but four times as much radius (remember that radius is proportional to velocity squared as previously covered.)

### Angle of Bank for Rate-One Turn

It is important for a pilot to know what angle of bank to fly for a rate-one turn. A rate-one turn is 3° per second or 360° per 120 seconds (two minutes.) You may have noticed that the turn coordinator in your aeroplane has “2 minutes” written on its face, which means that a rate-one turn takes two minutes for a 360° turn.

Time equals distance divided by velocity. For a rate one turn the distance is the circumference the circle the aeroplane is flying around in the time of 120 seconds. We can write:

$$T = \text{distance} / V$$

$$120 = 2\pi r / (V \times 6080/3600) \quad [6080/3600 \text{ needed to change } V \text{ to ft/sec if } r \text{ is in feet}]$$

Substituting the equation for r developed earlier and solving for tan(β) we get:

$$\tan(\beta)_{r1} = \frac{V}{364.13} \quad [\tan(\beta)_{r1} \text{ is for rate one turn. } V \text{ is in knots}]$$

The same equation can easily be developed by taking the rate of turn equation above and setting rate to 3, then solving for tan(B)

There is a rule of thumb used by pilots that says that the angle of bank for a rate one turn is:

$$\text{Bank for rate-one} = (10\% \text{ of TAS}) + 7$$

We now have two equations giving us angle of bank for rate-one. One equation is accurate but hard to use, the other equation is easy to use, but is it accurate?. Table 3 compares the results to the accurate equation to the rule of thumb:

TAS Knots	Bank By formula	Bank By rule of thumb
50	8	12
100	15	17
150	22	22
200	29	27
250	34	32

Note that the rule of thumb is not precisely accurate; having said that, it does come pretty close between 100 and 250 knots. Since that covers most light aircraft flight I recommend using the rule of thumb. To repeat the rule:

Bank for rate one =  $TAS/10 + 7$  [TAS is in knots]

### ***Radius of a Rate One Turn***

One of the most useful bits of information for pilots is to know the radius of a rate one turn. Pilots use this on many occasions, such as intercepting a DME arc, flying a holding pattern, or completing an IFR procedure turn. You may have heard that turn radius equals half of one percent of your true airspeed. Is this an accurate formula?

If you take the radius of turn equation:

$$r = \frac{V^2}{11.289 \tan(\beta)}$$

And substitute the rate-one turn equation:

$$\tan(\beta)_{r1} = \frac{V}{364.13}$$

You get:  $r = 32.26 V$  [This gives radius of turn in feet, for a rate one turn.]

To get radius of turn in units of NM divide by 6080, which gives:

$$r_{rate\ 1} = .0053 V \text{ [r in units of nautical miles]}$$

This is pretty much the same as say ½ of 1%. So the rule of thumb really is true. To be precise **radius of a rate one turn** is 0.53 of one percent of the true airspeed.

Diameter of a rate one turn is also important to know, and that of course is simply 2r, so diameter, in nautical miles, is just over 1% of the true airspeed.

## Drag

Just as lift is given by a lift equation, drag is given by a drag equation, which is:

$$D = C_D \times S \times \frac{1}{2} \rho V^2 \text{ [in metric units]}$$

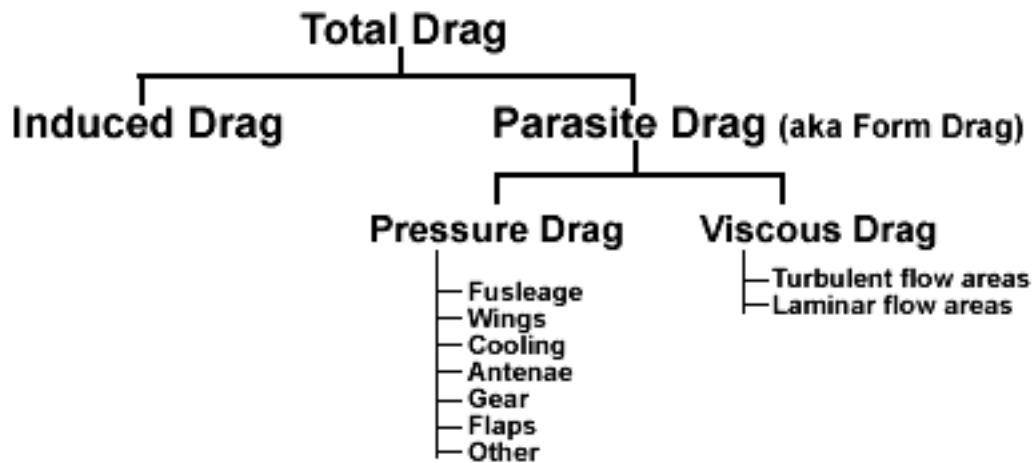
$$D = C_D \times S \times 1.426 \rho V^2 \text{ [D in pounds and V in knots]}$$

Notice that this equation is identical to the lift equation except that  $C_L$  has been replaced by a coefficient of drag  $C_D$ . Just as with coefficient of lift, coefficient of drag expresses the percentage of dynamic pressure turned into drag. We of course hope that the designer has kept  $C_D$  as low as possible; zero would be great. If that was the case there would be no drag and therefore we would need no thrust. Sadly no one has yet designed an aeroplane with zero drag.

The following analysis of drag grows a bit complex so it is important to keep in mind that ultimately drag is simply the  $P_s$  difference across a plane perpendicular to the flight path. In other words, if the pressure behind the aeroplane is less than the pressure in front of it there is drag. If we could create a situation where the static pressure in front and behind the aeroplane was the same, there would be no drag.

You might find it interesting to know that when aeronautical engineers measure drag in a wind tunnel it is usually *not* done with strain gauges, the way most pilots expect. Instead sensors are used to measure the speed of the air flowing just in front of the model and again just behind it. If the airflow has exactly the same speed in both locations then the dynamic pressure is the same in both locations and according to Bernoulli's equation the static pressure would also be the same, and there would be no drag. Unfortunately it always turns out that the velocity of the airflow behind the aeroplane is greater than that in front. Thus the **dynamic pressure is greater behind the aeroplane and the static pressure is lower**. That in a nutshell is what drag is. To reduce drag an aircraft designer must concentrate on keeping the speed of the air the same behind as ahead the aeroplane. In more common terminology, if the aeroplane "disturbs" the air (i.e. leaves it in motion) after the aeroplane has passed there is drag.

For the human mind it is useful to segment drag into parts that are due to different causes. This results in a hierarchy of drag as follows:



Induced drag is defined as drag due to the production of lift.

Parasite drag, or Form drag, is the remaining drag and is due to the combination of Pressure and Viscous drags.

We will now explore each of these drag types individually. But keep in mind that each contributes to the net static pressure difference across the plane perpendicular to the flight path. They don't really exist separately; it is simply a human convention to analyze them separately.

## Viscous Drag

Viscosity measures how sticky something is. Honey is viscous and water is less so. If you dip a spoon into honey and pull it out a lot of honey sticks to the spoon. If you put a spoon into water and pull it out some water sticks to the spoon, but less than with honey.

It may not seem like it to you but air has viscosity too - less than water and honey, but still there is some viscosity. Consequently there is some "sticking" of the air along the lateral edges of an aeroplane (sides of fuselage, and top and bottom of wing, etc.) as it passes through air. Accelerating this mass of air requires a force, as per rule 2, and thus creates drag, which we give the name viscous drag, or more commonly "skin friction."

Viscous drag is the dominant type of drag affecting aeroplanes in cruise (i.e. flying angle of attack less than  $5^\circ$ .) Viscous drag is confined to a layer of air immediately adjacent to the surface of the airplane. The layer within which all the viscous drag develops is called the boundary layer.

There is a relationship between the boundary layer, which is where viscous drag occurs, and pressure drag. We will get to that later, but first I must explain what a boundary layer is.

It is worth noting that viscosity of air *increases with air temperature*. I.E. air “sticks” to the sides of an aeroplane more on a warm day than a cold day. Therefore, there is more viscous drag on warm days.

It is also worth noting that the type of material the aeroplane is made of does *not* affect viscous drag. I.E. air does not stick any more or less to an aluminum aeroplane than a carbon-fiber or fiber-glass one. However some types of material are more suited to promoting laminar as opposed to turbulent boundary layers. That is the topic we will turn to next.

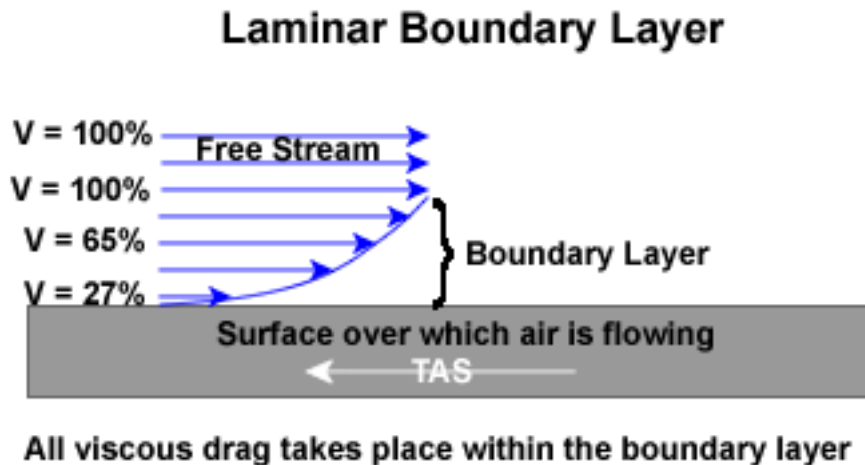


Figure 42

All the viscous drag occurs within a very thin layer of air just above the surface of the aeroplane. The thin layer of air within which the viscous drag develops is called the **boundary layer**. Figure 42 shows that at a distance equal to the width of one air molecule, the air sticks completely to the aeroplane. Since the aeroplane is moving through air that was originally stationary these air molecules are accelerated to the speed of the aeroplane and that requires a force (rule 2.) Viscous drag is the reaction (rule 3.) Luckily only a relatively few molecules are involved.

Figure 42 shows that the next layer of air molecules sticks a bit less, and the next layer sticks less, and so on, until a layer of molecules is reached that is not affected at all by viscosity. This represents the upper limit of the boundary layer. You are probably asking; how thick is a boundary layer? It is thicker on a slow aeroplane, and thinner on a fast aeroplane. It is thin near the leading edge of an object and gets thicker along the length of the object, as shown in Figure 43. On a typical light aeroplane the boundary layer is only two or three millimeters thick at the leading edge of the wing but more than a centimeter thick at the wing trailing edge (even thicker on the aft sections of the fuselage.) On a large aeroplane the boundary layer may be four or five centimeters thick at the trailing edge of the wing, yet less than one millimeter at the leading edge.

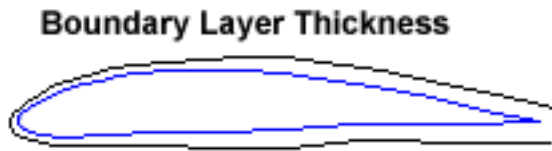


Figure 43

Figure 42 showed what is called a laminar boundary layer. Laminar simply means “in layers.” A boundary layer is called laminar when the velocity changes within it change uniformly and steadily as in Figure 42. In such a case the air molecules slip past each other smoothly, like hypothetical traffic on a multi-lane highway obediently following the rule that the right lane is slow, the next lane is faster, the next faster still, and so on.

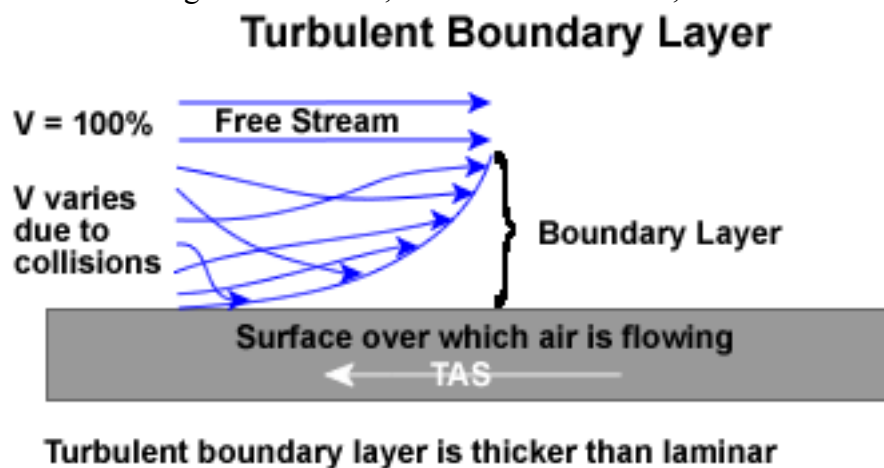


Figure 44

A turbulent boundary layer is more like traffic on a real highway. Drivers zigzag back and forth from lane to lane and no one can guarantee that the fastest car is always in the left lane. Often collisions, including rear-end pileups, can and do occur. In the case of air molecules the path each molecule takes is represented by a line, known as a streamline, as in Figure 44. A boundary layer where the streamlines cross frequently is called *turbulent*.

Turbulent boundary layers are always thicker than laminar boundary layers and have more kinetic energy, and thus they cause *more viscous drag*. It is *usually* desirable to have a laminar boundary layer. There are actually some special situations in which a turbulent boundary layer is preferable and we will discuss those in the section on pressure drag below. While a laminar flow is desirable it is nearly impossible to maintain the entire boundary layer in laminar flow. Natural laminar flow (NLF) is a major area of research in aerodynamics.

In most cases the boundary layer starts off laminar, but two things can and usually do cause it to become turbulent:



1. An obstruction on the aeroplane surface, comparable in size to the thickness of the boundary layer, will cause the boundary layer to become turbulent.
2. A *rapidly* slowing boundary layer will become turbulent.

Since the boundary layer is very thin near the leading edge of a wing or the nose of a fuselage any bugs, paint chips, rivet heads, etc. that protrude into the boundary layer will cause it to become turbulent, and thus increase viscous drag. Previously we said that the type of material the aeroplane is made of doesn't affect the amount of viscosity. But now we can see that smooth materials such as carbon fiber or fiberglass as opposed to riveted aluminum may be superior for promoting natural laminar flow and reducing viscous drag.

As we know, the air over the top surface of a wing accelerates to some maximum velocity and then slows down again. The slowing boundary layer is said to have a **negative velocity gradient**. If the negative velocity gradient is substantial the boundary layer will quickly become turbulent. This is pretty much the same as cars on a freeway that encounter a traffic slowdown. There may be collisions, and inevitably a lot of drivers change lanes as they try to keep speed up. In the case of airflow the inevitable result is a turbulent boundary layer. But this effect can be minimized if the change in velocity is *gradual*. Computer programs are used in modern aeroplane design to optimize the shape of the aeroplane (both airfoil and fuselage) so that there is as much natural laminar flow (NLF) as possible.

The amount of NLF can be maximized by:

1. Keeping the surface smooth. Good paint, wax, no rivets, etc.
2. Designing the aeroplane (wing, fuselage, fairings) so that the point of maximum air velocity is as far aft as possible.
3. Avoiding shapes that require substantial negative velocity gradients. Streamlined shapes, discussed under **Pressure Drag** below, approximately meet this criterion, but modern research is showing that subtle variations can make a significant difference in the amount of laminar flow.
4. Artificially inducing laminar flow (in which case it is not *natural* laminar flow.) This is not usually practical, but in theory using a pump to create a low-pressure area near the trailing edge, so that the boundary layer molecules never slow down, will do it.

Now that you know what viscous drag is you may be surprised to hear that the dimples on a golf ball are specifically designed to ensure the ball is surrounded by a turbulent boundary layer. This is because a laminar boundary layer generally causes more pressure drag, which is dominant in the case of a golf ball. Why? We will answer that question next.

## Pressure Drag

Pressure drag is the other component of parasite drag. It includes all the drag due to “disturbing” the air as the aeroplane passes through it. Usually each part of the aeroplane is analyzed individually, for example the fuselage, wings, gear, etc. each contributes pressure drag. Retracting the gear will eliminate gear-drag. Flap-drag disappears when flaps are retracted, etc.

Air passing through a piston engine compartment has heat added to it and usually exits in such a way that the airflow is disturbed. This is called cooling-drag. Equipping the engine compartment with cowl flaps will reduce it.

### Flow Separation at a Sharp Corner

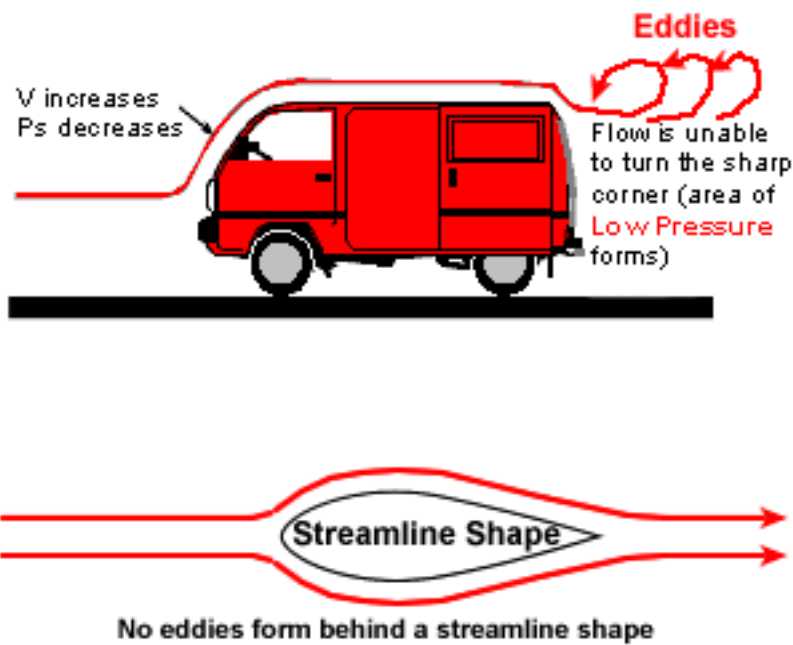
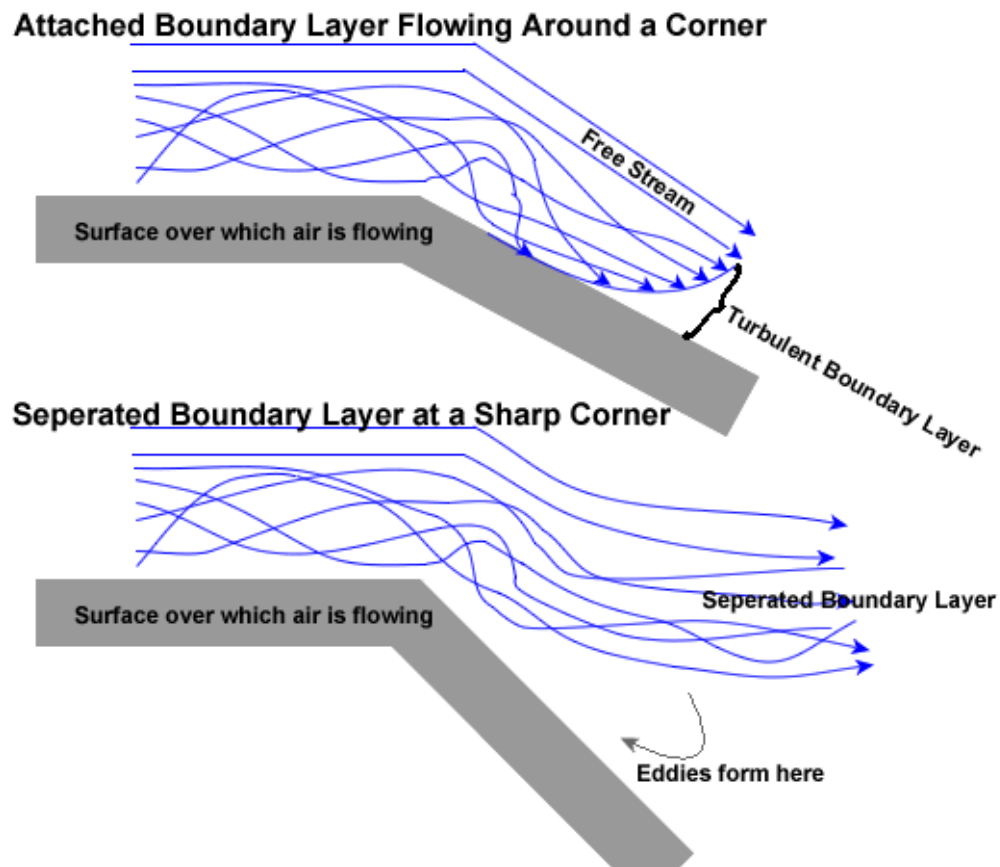


Figure 45

To minimize the drag from the fuselage and other components such as antennae the designer uses a technique called streamlining. To understand streamlining you must recall our previous discussion in **Lift Production in Subsonic Flow**. We know that air molecules can easily move aside as a subsonic aeroplane moves through them, but there is a limit to how quickly they can move, and that limit is the speed of sound. Consequently, if air flows past a sharp corner, as in Figure 45, it cannot adjust quickly enough and will be *disturbed*. **Eddies** will form downstream of the object.

Eddies are swirls of air left behind an aeroplane that has passed. They clearly represent a velocity that the air did not have before the aeroplane passed and therefore, as per Bernoulli's equation,  $P_s$  drops behind the aeroplane causing drag due to static pressure. That is why it is called *pressure drag*.



**Figure 46**

It is important to see that most pressure drag is due to a drop in pressure behind the aeroplane (because air is disturbed), as opposed to a rise in pressure ahead of the aeroplane. You can see visual indication of this by looking at all the dirty minivans driving around your hometown. It is always the back of the van that is dirty, compared to the front. The back of these vans get dirty because of eddies, from flow separation, which result in a low pressure area behind the van. The resulting inflow of air pushes dirt and deposits it on the back of the van. Of course these eddies also cause pressure drag, reducing the vans gas mileage.

At the leading edge of a wing (or any other object passing through the air) there is a small region where the air is stopped by the wing. This area experiences a high pressure (per Bernoulli's equation.) Since it is caused by stopping the airflow it is called the stagnation point. While this stagnation phenomenon is quite obvious, even intuitive, it is NOT the primary source of pressure drag, as explained in the preceding paragraph.

As previously explained, air passing a sharp corner cannot change direction fast enough to fill in behind the corner, and so becomes disturbed. It is worth asking, what happens to the boundary layer at that point? Figure 46 shows that the boundary layer separates from the aeroplane at a sharp corner.

**Boundary layer separation causes eddies** and therefore substantially increases pressure drag. Consequently there is an interaction between viscous and pressure drag, as mentioned earlier. If air had no viscosity it is obvious that there would be no skin friction, but since there would be no boundary layer the airflow reaching any sharp corners would have more kinetic energy and would expand around the corner rather than separate into an eddy, thus there would be no pressure drag either.

When a wing flies at larger angles of attack (greater than about  $5^\circ$  angle of attack) the pressure gradient on the aft section of the wing usually becomes severe enough to cause the boundary layer to separate near the trailing edge. Thus pressure drag begins to add progressively to the total parasite drag. By the time the wing reaches the stall pressure drag has become the dominant form of drag.

It is critical in aeroplane design to prevent the boundary layer from separating. In some cases this priority takes precedence over the desire to have a laminar boundary layer. If sharp corners are unavoidable then the designer will actually promote a turbulent boundary layer, because it has more energy and can better negotiate sharp corners. That is what the dimples on a golf ball are for. They make the boundary layer turbulent and more likely to stay attached. Figure 47 shows two balls, one with dimples and one without. Eddies behind the smooth ball are larger so there is more pressure drag. The dimpled ball has slightly more viscous drag, but less pressure drag.

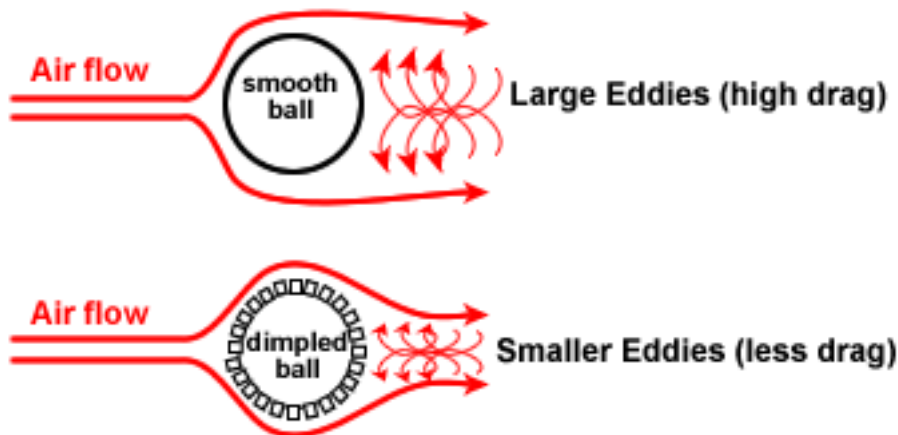


Figure 47

Any round balls, even ones with dimples, disturbs the airflow significantly because the air cannot fill in behind quickly enough. A better shape is the teardrop design shown in Figure 45. This is called a “streamline shape” because streams of air molecules can follow each other around it in a smooth fashion. (A streamline is the path that a series of air particles follow. But an object is “streamlined” when the streamlines around it are smooth, i.e. with no eddies.)

A lot of research is done into optimizing streamlining, but the basic idea is a round leading edge with a sharp trailing edge. Consequently wings are inherently streamlined, But fuselages, antennae, wing struts, gear legs, etc. need to be made streamline to

minimize pressure drag. Surrounding each item with a streamline fairing does the trick. Note that even though the resulting shape may be larger it will produce less drag if it minimizes eddies. Note also that fairings increase viscous drag while they reduce pressure drag – but the trade off is worth it.

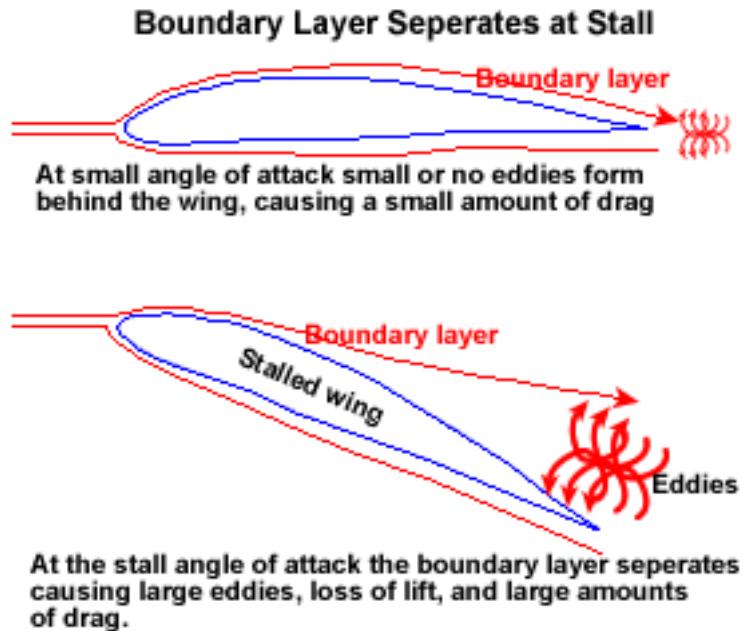


Figure 48

In our previous discussion about  $C_{Lmax}$  we learned that there is a particular angle of attack (the **stall** angle of attack) beyond which lift begins to drop off. Pressure drag also dominates at and beyond the stall angle of attack. You should now suspect that eddies begin to form as the stall is approached and eventually dominate the flow. Consider Figure 48, which shows a wing flying at high angle of attack. At high angle of attack air is accelerated a lot over the wing. At some critical angle of attack, the negative velocity gradient on the top surface of the wing becomes too much for the boundary layer (refer back to the discussion about why the boundary layer becomes turbulent when velocity is decreasing and carry the logic there through to this more extreme case.) There is an angle of attack where the boundary layer not only separates but actually reverses, as shown in Figure 48. Once this happens eddies form, pressure drag increases markedly, and lift decreases.

### Interference Drag

As mentioned previously, aircraft designers analyze the drag of each component of an aeroplane, attempting to optimize (streamline and induce NLF) each. But, when you mate the parts together there is always more drag than the sum of each part. This phenomenon is due to the interference of boundary layers at the junctions. If one boundary layer causes

another to separate the result is a substantial increase in *pressure drag*, this is called interference drag.

Interference drag can be minimized by:

1. Reducing the number of junctions. For example V-tail rather than conventional tail. Monoplane rather than Bi-plane, etc.
2. Making the junction at an angle more than 90° if possible. Acute angles bring the boundary layers closer together and increase interference.
3. Installing filets. A filet is a smooth fairing that allows two boundary layers to merge gradually. Figure 49 shows a typical filet.

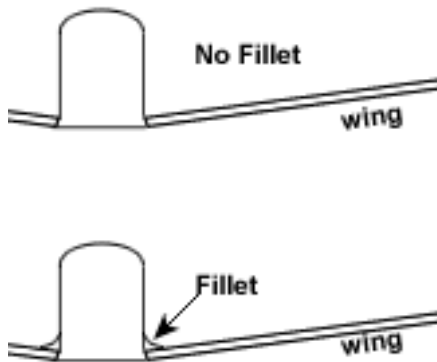


Figure 49

## Parasite Drag Equation

Parasite drag is the sum of viscous and pressure drag. Viscous drag dominates at small angles of attack and pressure drag at higher angles of attack.

Parasite drag ( $D_p$ ) can be calculated using an equation very similar to the lift equation:

$$D_p = C_{Dp} \times S \times \frac{1}{2} \rho V^2 \quad [\text{metric units}]$$

$$D_p = C_{Dp} \times S \times 1.426 \rho V^2 \quad [\text{in our usual units}]$$

$C_{Dp}$  is called the coefficient of parasite drag. Its value depends purely on the shape of the aeroplane. In other words it depends primarily on how streamlined the aeroplane is. Table 3 gives a few sample  $C_{Dp}$  values (these are representative only and are not for any specific sub-model of the type.)

aeroplane	$C_{Dp}$ (clean)
C-172	0.047

Lear Jet	0.030
A 320	0.022 <sup>14</sup>

Table 3

$C_{Dp}$  is a *fixed value* that changes only when the shape of the aeroplane changes. In other words it changes when gear is extended or retracted, or when flaps are extended or retracted, or when cowl flaps are opened and closed.  $C_{Dp}$  does *not* change when speed or angle of attack change.

A windmilling propeller, as experienced after an engine failure, increases  $C_{Dp}$ .

Ice, bugs, and other contaminants on an aeroplane increase  $C_{Dp}$ .

### Equivalent Flat Plate Area

Some text books use an alternative to the coefficient of parasite drag known as the equivalent flat plate area (f.) By definition  $f = C_{Dp} \times S$ . Therefore  $D_p = f \times 1.426V^2$ . The advantage of using equivalent flat plate area is that the value captures the total parasite drag more effectively since larger aeroplanes always have large values of f. The disadvantage is that it does not capture the relative efficiency of the design the way  $C_{Dp}$  does, so it will not be used in this text.

### Induced Drag

Induced drag is *drag due to the production of lift*. Many pilots are confused by this definition and often mistakenly think that induced drag is drag caused by the parts of the aeroplane that create lift, namely the wings. This is not so. The wings create parasite drag (both viscous and pressure, as explained above) and on some aeroplanes the fuselage contributes to lift, if it is shaped so that air accelerates over the top of the fuselage, thus induced drag is *not caused by any particular part of the aeroplane* it is simply drag that results when lift is created.



Figure 50

<sup>14</sup> Later we will learn that  $C_{Dp}$  does increase above a critical Mach number

Figure 50 shows the “sheet” of air particles “pouring” off the trailing edge of a wing<sup>15</sup>. Rule 3 tells us that these particles are deflected downward and lift is the reaction upward. Figure 50 shows a hypothetical 2-dimensional flow. If the situation were actually 2-dimensional, as drawn in this figure, there would be lift but no induced drag. Induced drag results from changes to the shape of this sheet due to the wingtip vortex. The vortex adds a span wise component to the airflow giving it a 3-dimensional shape.

### Three Dimensional Flow – The Wing Tip Vortex

Figure 51 shows the wing of an aeroplane as viewed from the rear.  $P_s$  is lower above the wing than below **if** lift is being produced. The more lift produced the greater the difference in  $P_s$ .

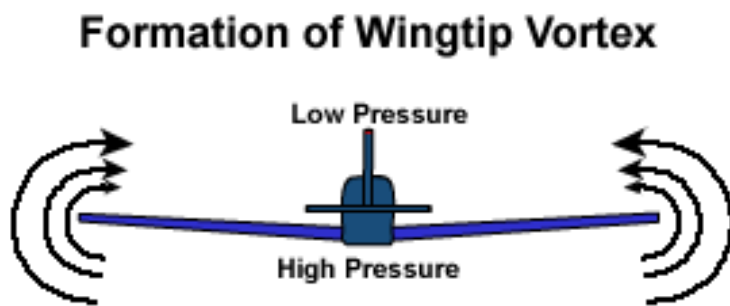


Figure 51

At the wingtip the high-pressure air below the wing can, and does, escape around the wingtip. This has the following effects:

1. A wingtip vortex forms, rotating air from below the wing to above the wing.
2. Air within the trailing sheet moves outboard below the wing and inboard above the wing, leaving small vortices behind the full wingspan, but stronger near the tip.
3. The vortex reduces static pressure difference between the upper and lower wing surfaces; therefore less lift is produced, especially near the wingtip.
4. The trailing sheet is depressed near the wingtip, as shown in Figure 52. This causes a flow *circulation* near the wingtip. Circulation is explained below.

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<sup>15</sup> To visualize the sheet imagine a decorative waterfall as found in many office buildings. Alternatively tape a sheet of paper to the wing-trailing-edge on a model aeroplane.





Figure 52

The wingtip vortex is potentially dangerous. The vortex from a large aeroplane can easily throw a small aeroplane out of control. The vortex is strongest when the aeroplane is at maximum weight and maximum angle of attack, for example just after takeoff and during initial climb. Light aeroplanes should never takeoff behind heavy aeroplanes until sufficient time has elapsed for the wingtip vortex to dissipate. Dissipation will happen faster with a strong wind to break up the vortex. On a calm day the vortex will persist for many minutes.

A wingtip vortex reduces the amount of lift a wing produces. For a straight-rectangular wing the loss is greatest near the wingtip (all wings produce a bit less lift than they would without the vortex.)

As the vortex curls back on itself it deflects the sheet of air particles coming off the trailing edge of the wing as shown in Figure 52. The air particles behind the wing, especially near the wingtip, are deflected downward more than they would be without the vortex. This might sound like good news, but unfortunately it results in a pressure field forming that causes a phenomenon known as *circulation*. Consider Figure 53. The diagram shows that air particles ahead of the wing rise before the wing reaches them, and that is bad news. Why does it happen?

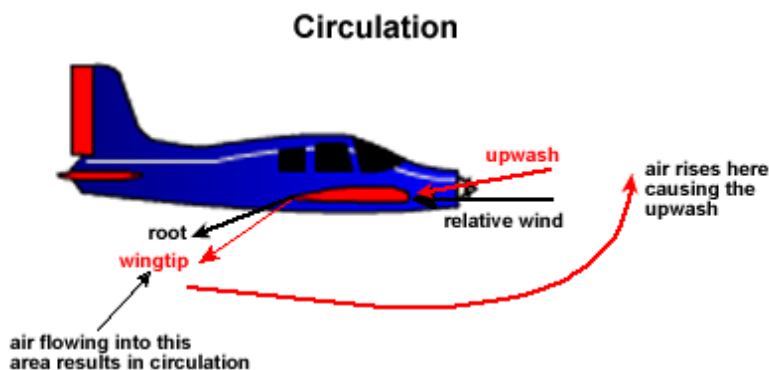


Figure 53

Figure 53 shows that deflecting air down behind the wing tends to increase static pressure there, but as we know air resists compression, thus a pressure wave moves air particles through the air creating a “circulated” flow as shown. This circulation allows air density to remain uniform around the wing but causes the aeroplane to experience a locally deflected relative wind. Circulation causes the air ahead of any subsonic-wing producing lift to rise slightly, as shown and the need to “climb” the up-wash, even in level flight, result in induced drag.

It should be clear that circulation can happen only if the wing flies slower than the speed of sound (because the circulation is limited to that speed). There will be no induced drag in supersonic flight.

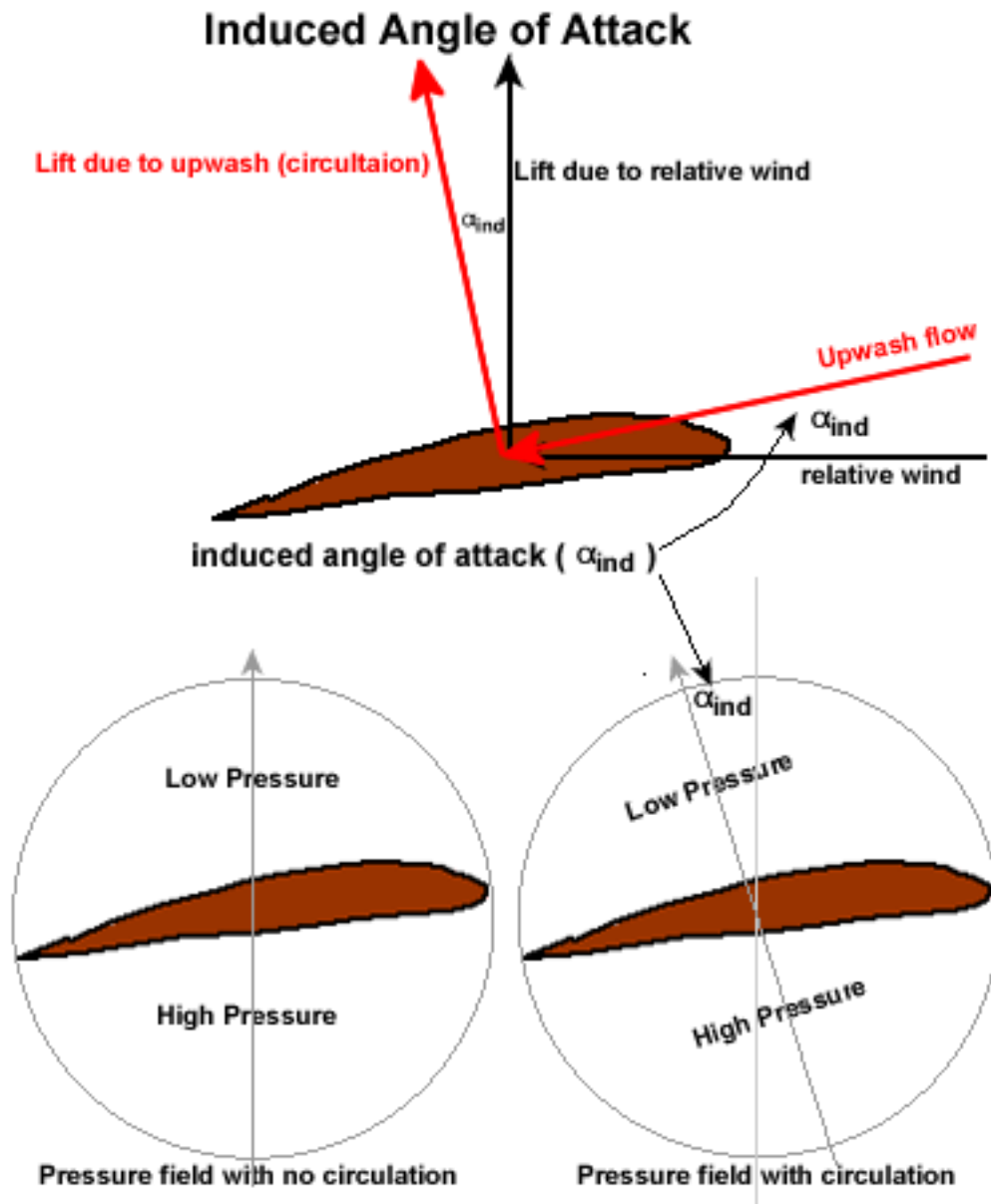


Figure 54

The circulation of airflow is responsible for induced drag. It *rotates* the *lift creating* pressure field slightly. The lower left corner of Figure 54 shows how the pressure field would look if there was no circulation. There would be a  $P_s$  difference across the flight path (i.e. lift) but no pressure difference across the plane perpendicular to the flight path, so there would be no induced drag. This is approximately the situation near the wing root of a rectangular wing.

The lower right corner of Figure 54 shows how the pressure field looks when there is circulation. Most  $P_s$  difference is still across the flight path (i.e. lift) but a small  $P_s$  difference now exists across the plane perpendicular to the flight path, so there is a small amount of induced drag ( $D_i$ .) This is the situation near the wingtip of a rectangular wing.

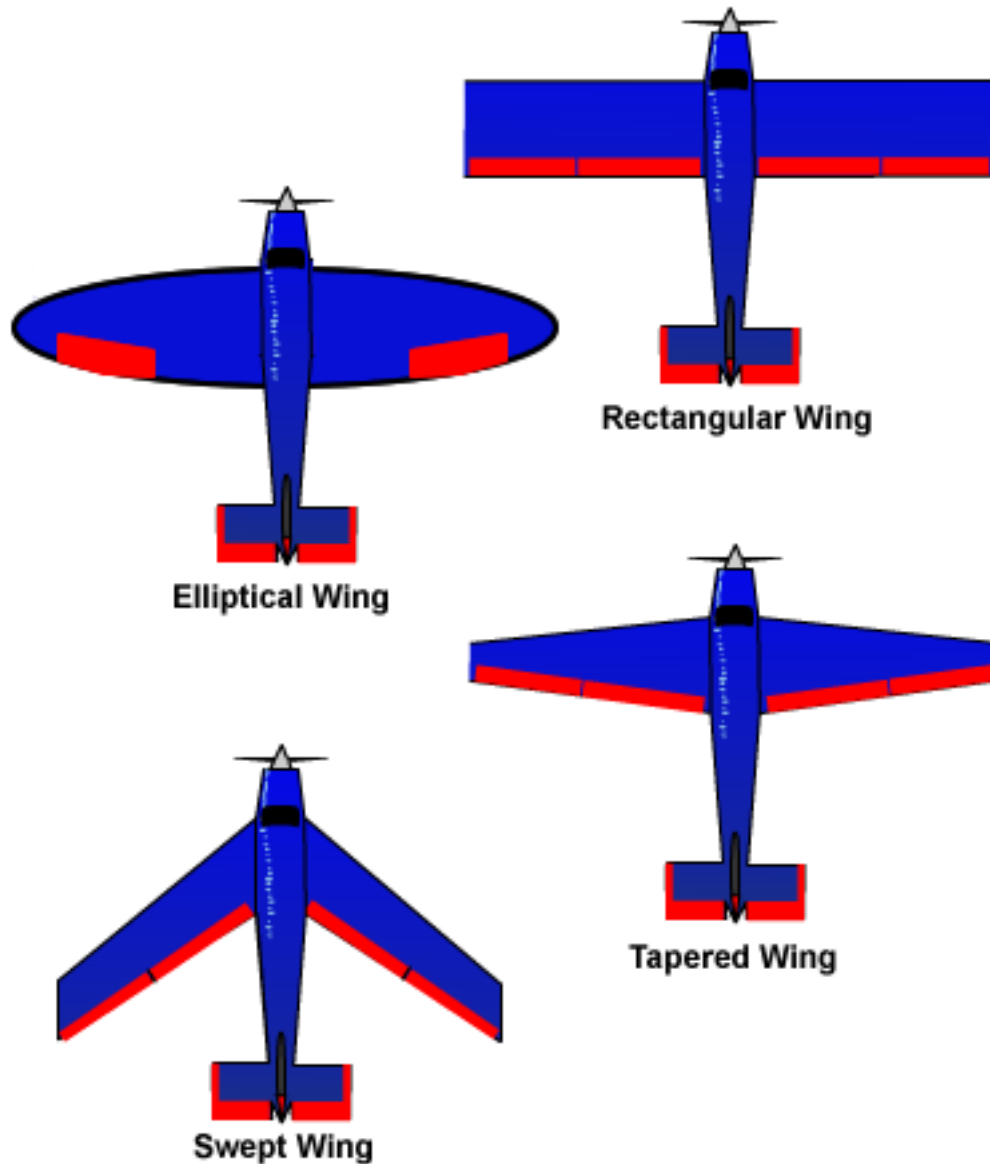
The angle that the pressure field is rotated, shown in Figure 54, is called the induced angle of attack ( $\alpha_{ind.}$ ) Previously we defined angle of attack ( $\alpha$ ) as the angle between the chord and relative wind. However the wing actually experiences a lesser effective angle of attack ( $\alpha_{effective}$ ) which is equal to  $\alpha$  minus  $\alpha_{ind.}$

$$\alpha_{effective} = \alpha - \alpha_{ind.}$$

There is a benefit of this in that the wing will not stall until  $\alpha_{effective}$  reaches the stalling angle of attack. Since  $\alpha_{effective}$  is always less than  $\alpha$  stall is delayed near the wingtip. We will explore this effect in detail later.

The greater  $\alpha_{ind}$  the more induced drag there is. On rectangular-wing aeroplanes this drag is concentrated at the wingtips, on swept wing aeroplanes it spreads to the root. We will now explore what factors determine the amount of induced drag.

The most important factor determining induced drag is also the most obvious. With greater angle of attack more circulation develops therefore induced drag is greatest at large angles of attack and less at small angles of attack. At the zero-lift angle of attack there is no circulation, and no induced drag.



**Figure 55**

The other factor that determines induced drag is a bit less obvious. It is the shape of the wing, which we call its planform. By planform we mean the aspect ratio, taper ratio, and sweep of the wing. Figure 55 shows several different wing shapes ranging from a simple rectangle, a straight tapered wing, a swept wing, and an elliptical wing. There are an infinite number of possible wing shapes of various aspect ratios and sweep angles. (Review aerodynamic terminology on page 15.) Why is planform shape a significant factor affecting induced drag?

Since induced drag is created by the wingtip vortex it makes sense that a short wing (one with low aspect ratio) will be more severely afflicted and an infinite span wing would have no induced drag at all. High aspect ratio wings, such as those found on gliders,

produce minimum induced drag while low aspect ratio wings such as those found on jet fighters produce a lots of induced drag (if flown at high angle of attack.)

So far we have said that on a rectangular-wing induced drag is concentrated near the wingtip. Is it possible to shape a *straight* wing such that induced drag (circulation) is uniform along the wingspan? Yes, it is possible.

Consider Figure 56, which shows the lift distribution along the wingspan of two wings. The upper wing is rectangular, and you can see that lift is fairly high and uniform near the root but drops off rapidly near the wingtips. Obviously the vortex on this wing concentrates its affect near the wingtip and induced drag is also concentrated there. An example of such an aeroplane is the Piper Cherokee 140. Since the lift distribution does not match the shape of the wing neither lift nor induced drag are distributed uniformly on this aeroplane.

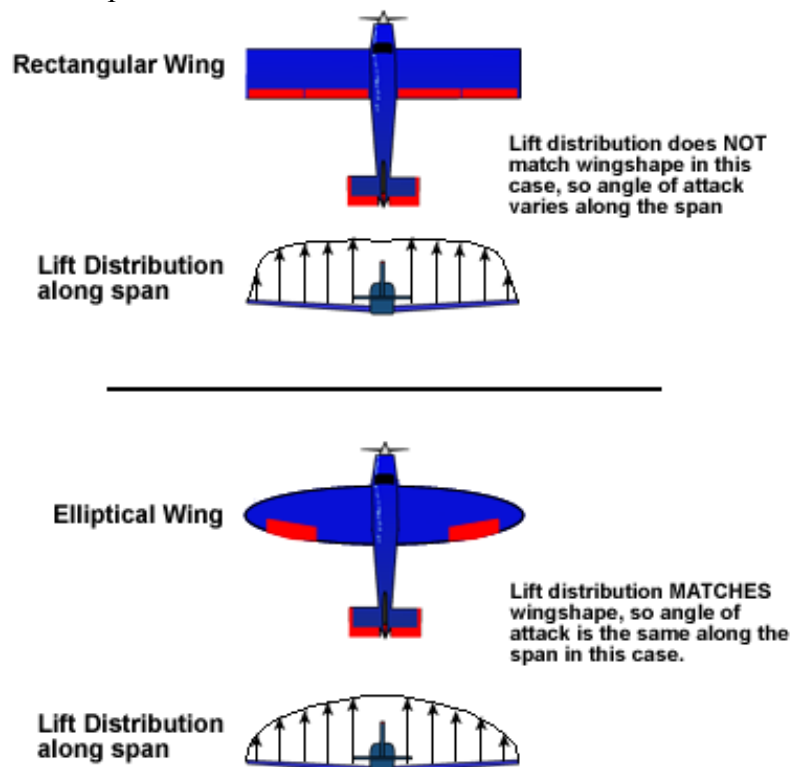


Figure 56

The second wing in Figure 56 has an elliptical lift distribution. The lift is maximized at the centerline and drops off exponentially toward the wingtips. If the shape of the wing exactly matches the lift distribution then both lift and induced drag are uniformly distributed along the wingspan (i.e. static pressure difference between top and bottom of the wing is the same everywhere.) An elliptical wing satisfies this requirement. It is the only shape that does. (A straight-wing of aspect-ratio six and taper-ratio 0.5 with a round wingtip fairing is a reasonable approximation (and cheaper to build.)) Take note that an

elliptical wing does not escape induced drag it simply spreads it uniformly from wingtip to wingtip, unlike the rectangular wing, which concentrates it near the wingtips.

Swept wings increase the amount of circulation the vortex induces, especially near the wing root (see Figure 57.) Consequently swept wing aeroplanes experience significantly more induced drag. Because the depression of the trailing sheet spreads inboard the root of the wing may have more induced drag than the tip. This adversely affects stall characteristics as explained below.

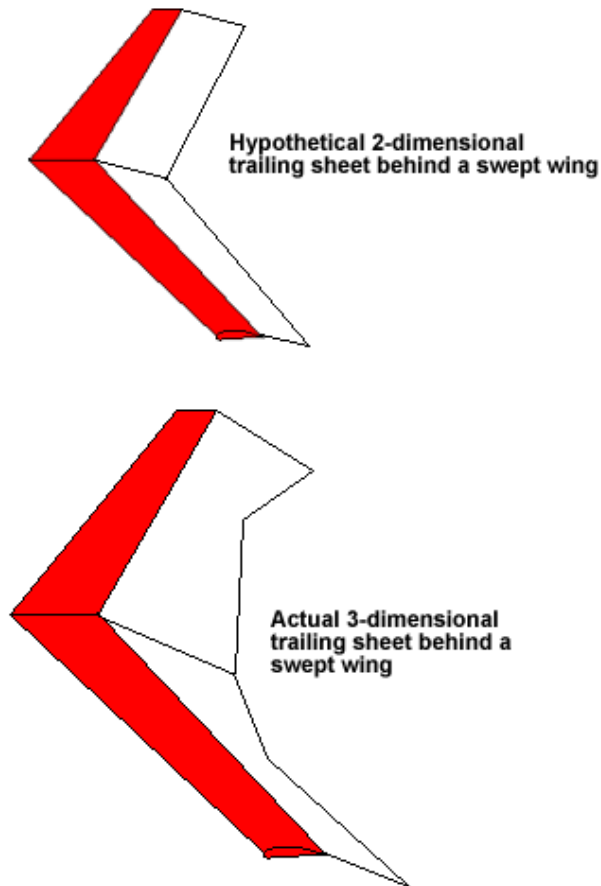


Figure 57

## Wing Stall Patterns

We previously learned that a wing stalls at a particular angle of attack. But we now know that circulation is not distributed evenly along a wingspan. We should suspect that stalling angle of attack is not the same at every station along the wingspan.

$$\alpha_{\text{effective}} = \alpha - \alpha_{\text{ind}}. \quad [\text{Wing will stall when } \alpha_{\text{effective}} = \alpha_{\text{stall}}]$$

For a rectangular wing  $\alpha_{\text{ind}}$  is greatest near the wingtip and  $\alpha_{\text{effective}}$  is less there. In other words the vortex on a straight wing delays stall at the tip and causes the root to stall first.

This is a nice safety feature because it is safer if the root stalls first because the ailerons are near the wingtip; so delaying wingtip stall improves aileron control during a stall.

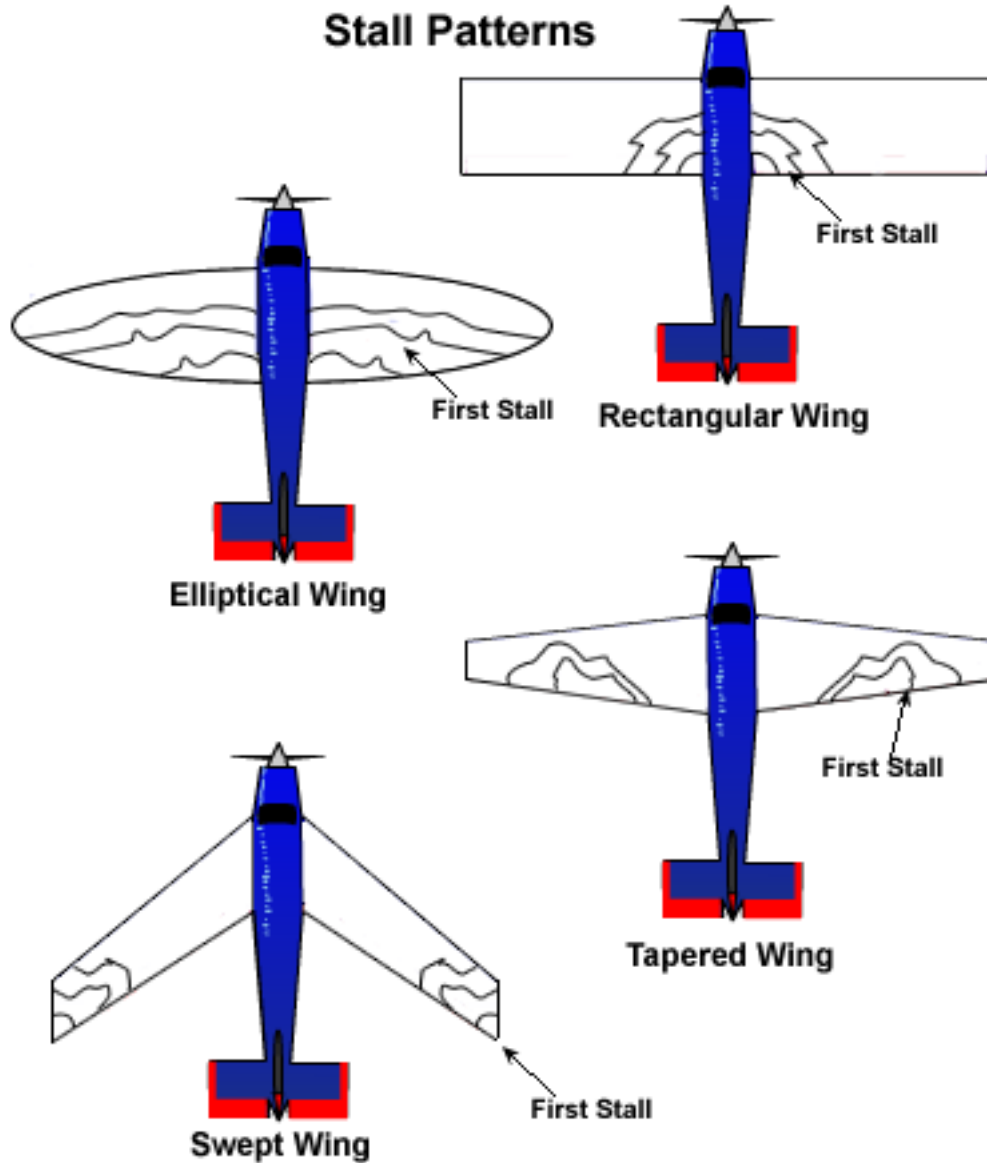


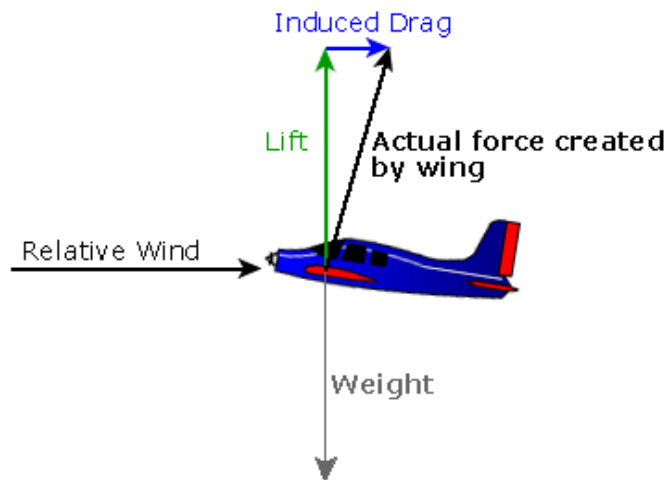
Figure 58

Figure 58 shows stall patterns. The marked areas show where the stall will first occur. The rectangular wing is most desirable. The elliptical wing is undesirable as the entire wing stalls at once (because  $\alpha_{ind}$  is the same all along the wing, thus  $\alpha_{effective}$  is the same everywhere.)

A tapered and swept wing has a particularly dangerous stall characteristic as the wingtip stalls first, causing a loss of aileron control<sup>16</sup>. Fortunately the professional pilots who fly such aeroplanes seldom stall them<sup>17</sup>.

Undesirable stall patterns, i.e. tip stalling first, can be improved by *wash out* to. **Washout** means to twist the wing so that the tip is at a lesser angle of attack than the root. This ensures that the root stalls first allowing the ailerons to work right through a stall, which is much safer. This design feature is not needed on a rectangular wing but is commonly employed on tapered wings.

## Induced Drag Equation



**Figure 59**

Figure 59 shows that induced drag is the component of the static-pressure-field (labeled as “Actual force created by wing” in the diagram) parallel to the relative wind. The other component is lift. Lift and induced drag go hand in hand and that is why we say that induced drag is due to the production of lift.

Induced drag ( $D_i$ ) can be calculated using an equation very similar to the parasite drag equation:

$$D_i = C_{Di} \times S \times 1.426\rho V^2$$

$C_{Di}$  is called the coefficient of induced drag. Its value depends upon angle of attack ( $C_L$ ) and aspect ratio (AR.) It is given by the formula:

$$C_{Di} = C_L^2 / (\pi e AR)$$

<sup>16</sup> Tip stall on a swept wing aeroplane may also cause a nose-up pitch that deepens the stall, making the situation even worse.

<sup>17</sup> Many jet airliners have safety features built in that virtually guarantee that they will not stall. For example a “stick pusher” that takes over and pushes the control column forward if angle of attack gets too close to the stall.



In the above equation  $C_L$  is the coefficient of lift, which as we know varies with angle of attack.  $\Pi$  is pi, the Greek letter representing 3.1415... Theoretical calculations show that  $\Pi$  would be the correct constant if  $\alpha_{\text{effective}}$  was the same at all stations along the wingspan, i.e. if lift distribution was elliptical. In most cases it is not however, so a second constant known as the Oswald efficiency (e) number is added to the equation. We must think of e as expressing the quality of span wise lift distribution. If the distribution is perfectly elliptical then  $e=1.0$ . In most real world cases e is less than one. Typical values of e range from 0.7 to 0.9 for most aeroplanes.

AR is the aspect ratio ( $b^2/S$ .) We should not be surprised to see this factor in the equation. It expresses the fact that a longer wingspan will substantially reduce induced drag. In fact doubling the wingspan cuts induced drag to one quarter, because span is squared in the AR definition.

The induced drag equation as given above is not very useable because  $C_{Di}$  keeps changing as angle of attack changes. But we learned previously in the section “Coefficient of Lift for Straight Flight” on page 45 that one unique value of  $C_L$  is associated with each velocity. Substituting the  $C_L$  equation into the  $C_{Di}$  equation above we get an equation for induced drag **in un-accelerated flight** of:

$$D_i = W^2 / (1.426 \Pi e \rho b^2 V^2)$$

Don't attempt to memorize this equation, instead examine it for what you can learn about induced drag. The things you can learn are described in the next three paragraphs.

When air density decreases induced drag increases, therefore induced drag is more of a problem for high flying aeroplanes.

As span (b) increases induced drag dramatically decreases. Doubling span cuts induced drag to one quarter. Consequently, expect high flying or slow aeroplanes to have long wingspans.

Induced drag is inversely proportional to velocity squared, so slow aeroplanes experience more induced drag than fast aeroplanes (other things being equal.) As any aeroplane slows induced drag increases dramatically.

Figure 60 shows a graph of parasite and induced drag plotted against velocity.

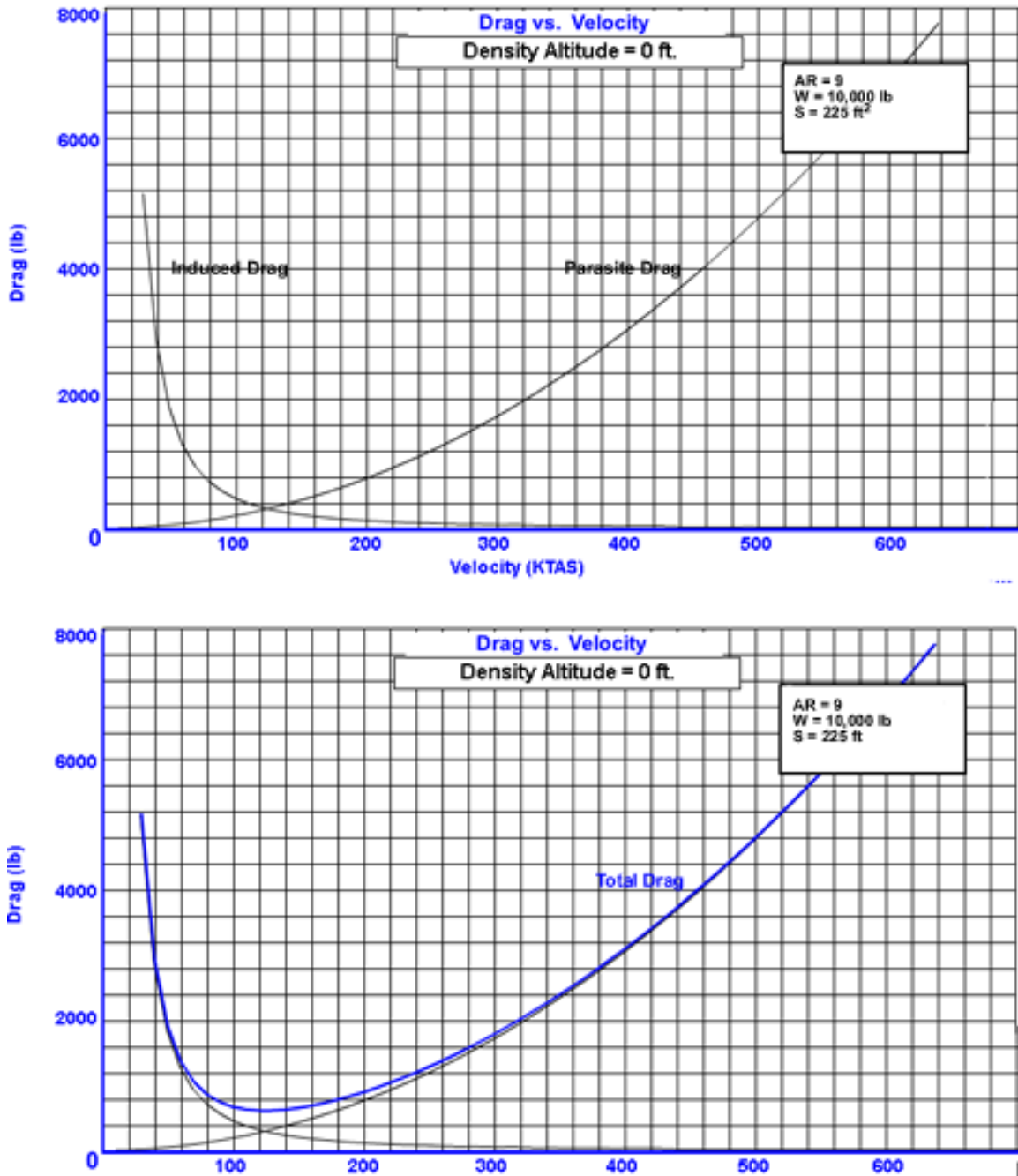


Figure 60

## Ground Effect



Ground effect is a phenomenon that most pilots have heard of but which often confuses them. A common image is that of a “cushion” of air between the wing and the ground. This image is completely false.

Ground effect occurs because at low altitude the ground blocks the wingtip vortex that creates circulation and induced drag. If the air cannot circulate around the wing, as described earlier, then there is no induced drag and no induced angle of attack ( $\alpha_{ind.}$ )

Figure 61 shows  $C_L$  vs.  $\alpha$  graph for a wing in and out of ground effect. Earlier we saw that the vortex reduces the amount of lift a wing produces and we can see that the loss is recovered in ground effect. But, to fully block the vortex a wing must fly almost touching the ground. Once altitude is above the radius of the vortex there is no ground effect.

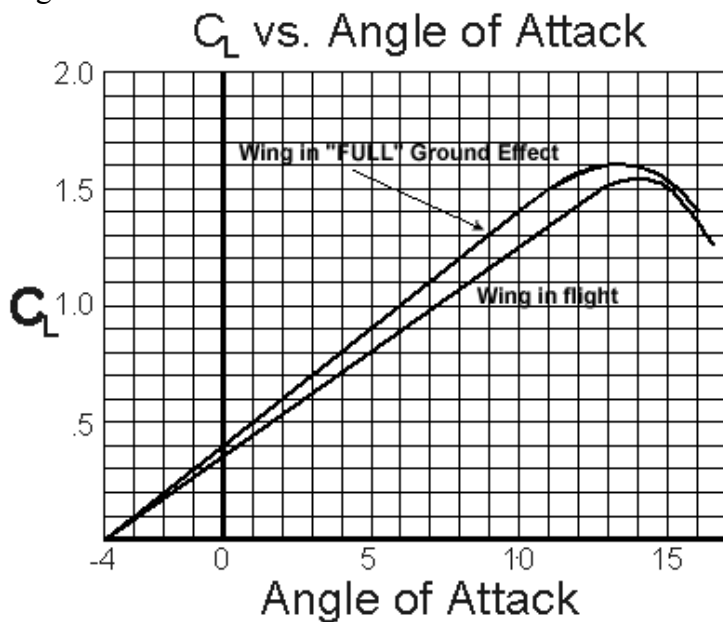


Figure 61

Figure 62 shows that *measurable* ground effect extends to about  $\frac{1}{2}$  wingspan above ground and is therefore normally encountered only on takeoff and landing. While there is a slightly increased lift in ground effect the main result is decreased induced drag.

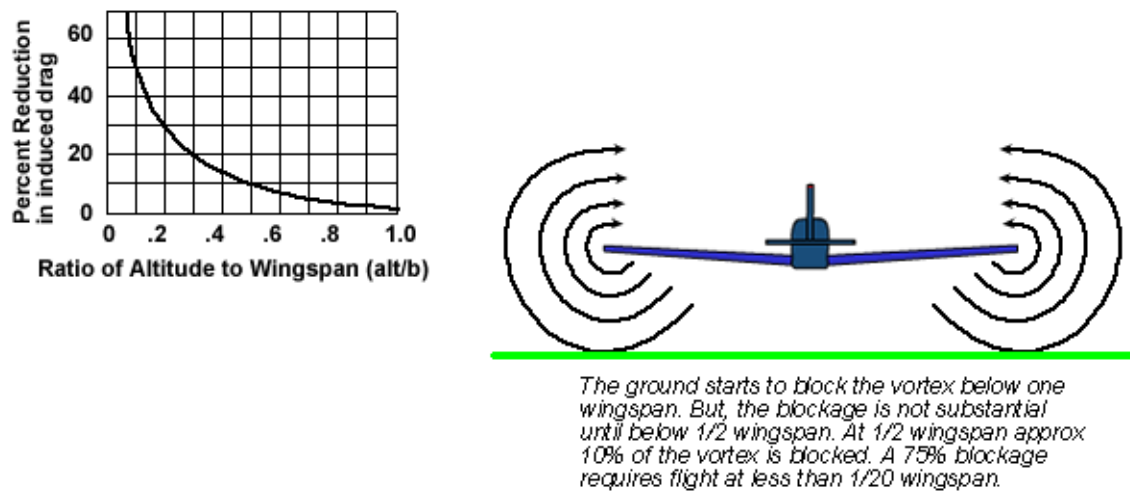
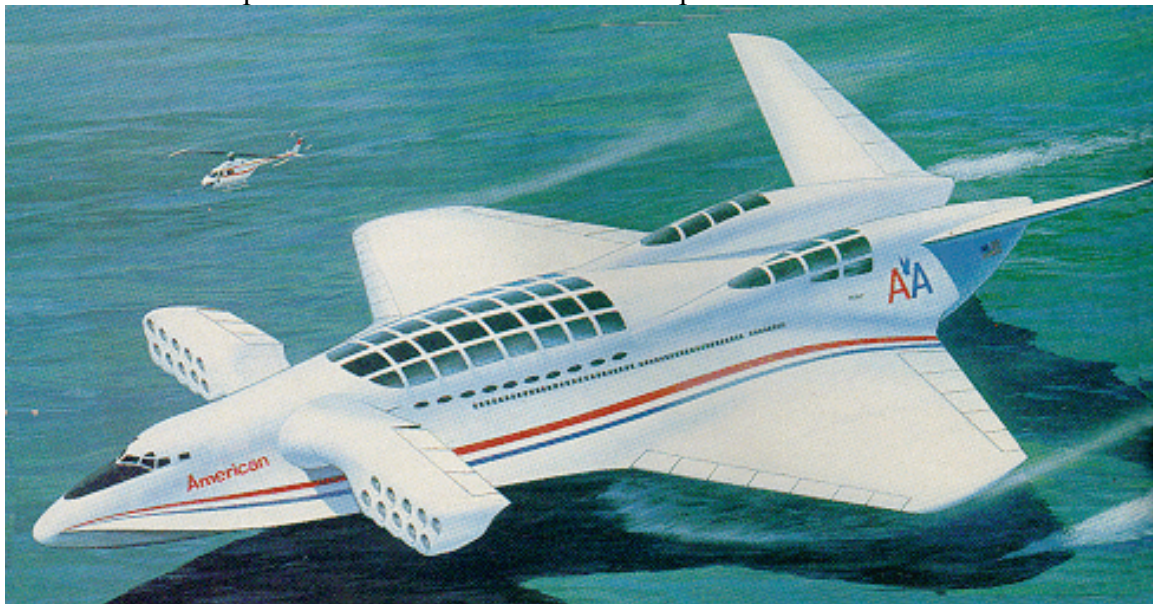


Figure 62

Reduction of induced drag when in ground effect makes it possible to accelerate more quickly after lift off, but the aeroplane must remain in ground effect. This procedure is commonly taught in soft filed operation where the takeoff involves lift-off at high angle of attack such that induced drag would be substantial if the aeroplane left ground effect.

On landing ground-effect s prolongs the “floating” tendency of an aeroplane. This is usually not desirable. Consequently aircraft designers sometimes modify the wingtip shape to increase or decrease ground effect. For example drooping the wingtips pushes the vortex closer to the ground thus improving takeoff performance, but increasing float on landing. Curving the wingtip upwards has the opposite effect. This later design is called a Hoerner tip and is common on Beech and Piper aircraft.



*Artist's rendition of a futuristic "WIG SHIP" (Wing In Ground effect.) W = 10 million pounds, cruise altitude = 100 feet asl.*

## Winglets

Winglets are devices added to wingtips that might be said to follow the old philosophy, "if you can't lick them, join them." The winglet *does not* block the vortex, it uses it. The inboard flow of air above the wingtip can be used to create thrust. The stronger the vortex the more thrust is generated. With a winglet, as induced drag increases the offsetting thrust also increases, counter balancing induced drag to a substantial degree.



Winglets are in common use on high flying jet transport aeroplanes. Sadly they are also sometimes employed on aeroplanes that would be better off without them<sup>18</sup>. Winglets are only called for on *highflying* aeroplanes that operate at *significant angles of attack*. Light aeroplanes that normally fly low and at small angle of attack do not benefit from winglets. To be warranted a winglets reduction in induced drag must be more than the increased parasite drag it creates.

## Total Drag

Total drag is the sum of parasite and induced drag.  $D = D_p + D_i$

Figure 63 shows a graph of total-drag vs. velocity in un-accelerated flight. It is important to remember the distinctive U-shape form of this graph. It shows that at slow speed induced drag dominates, but at high speed parasite drag dominates.

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<sup>18</sup> Winglets became somewhat of a fashion in the 1980s. It is hard to overestimate the power of marketing. For a while it seemed that an aeroplane could not be sold unless it had winglets, whether aerodynamically justified or not.

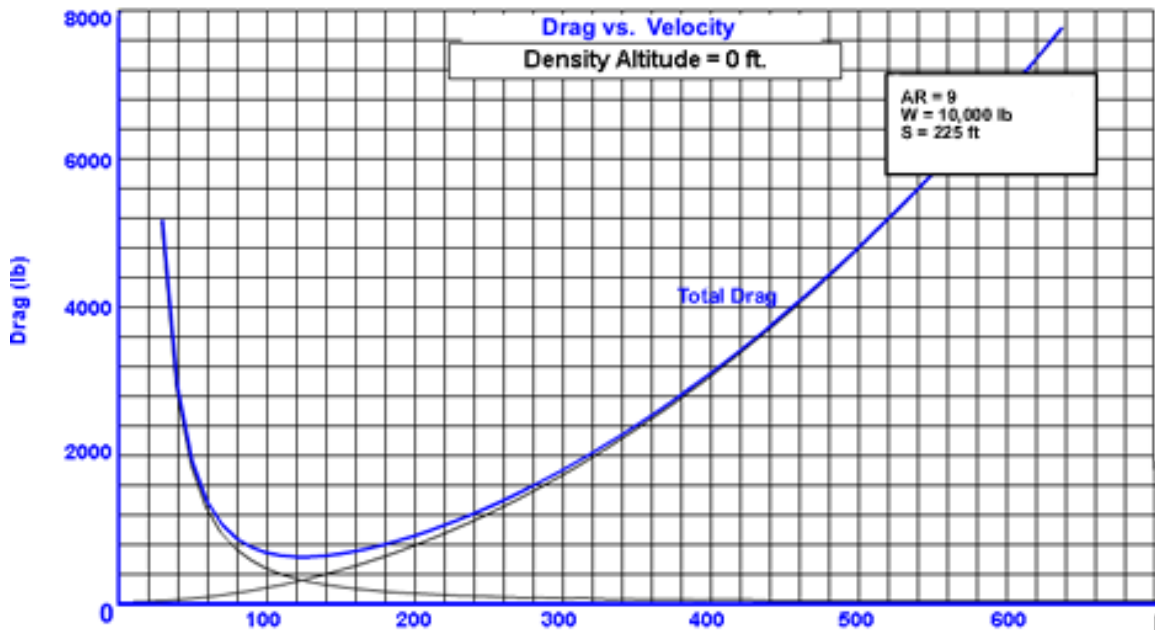
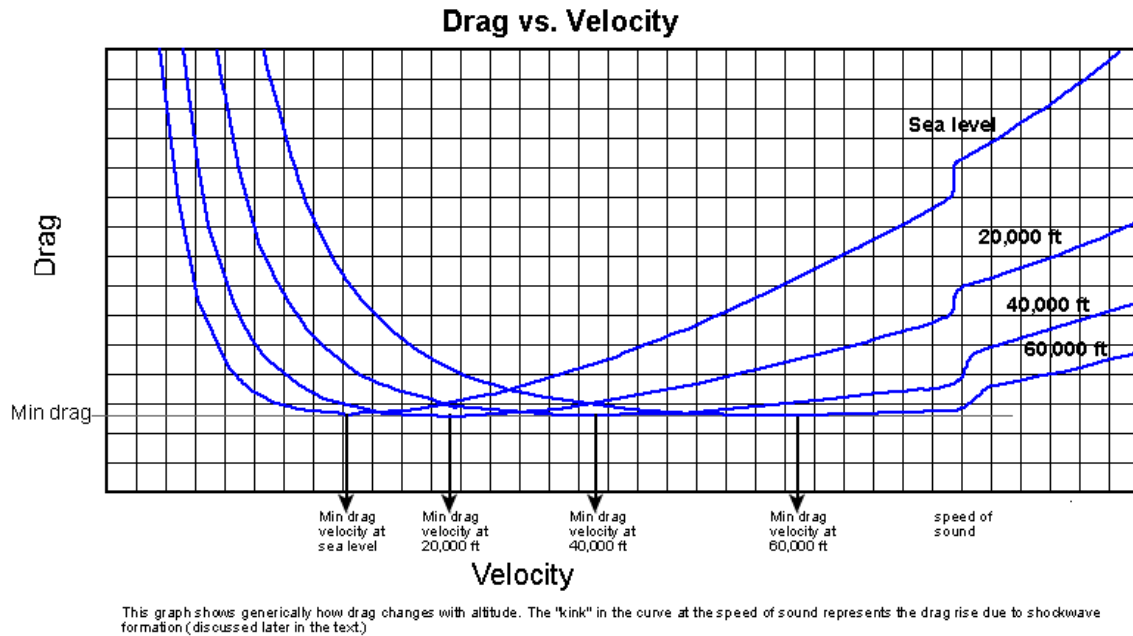


Figure 63

The bottom of the “U” is the speed for minimum drag. All flight slower than that speed (left side of graph) is said to be on the “back side” of the drag curve.

Can you predict how the Drag vs. Velocity curve changes at higher altitudes? Parasite drag will decrease, as indicated by the equation:  $D_p = C_{Dp} \times S \times 1.426\rho V^2$  ( $\rho$  decreases as altitude increases.) But induced drag increases with altitude in accordance with the equation:  $D_i = W^2 / (1.426 \Pi e \rho b^2 V^2)$ . The result is shown in the figure below. There are two important things to notice:

1. The drag curve moves to the right with altitude (i.e. the aeroplane can fly faster for a given amount of drag at altitude.)
2. The bottom of the curve, representing the minimum possible drag **does NOT change** with altitude.

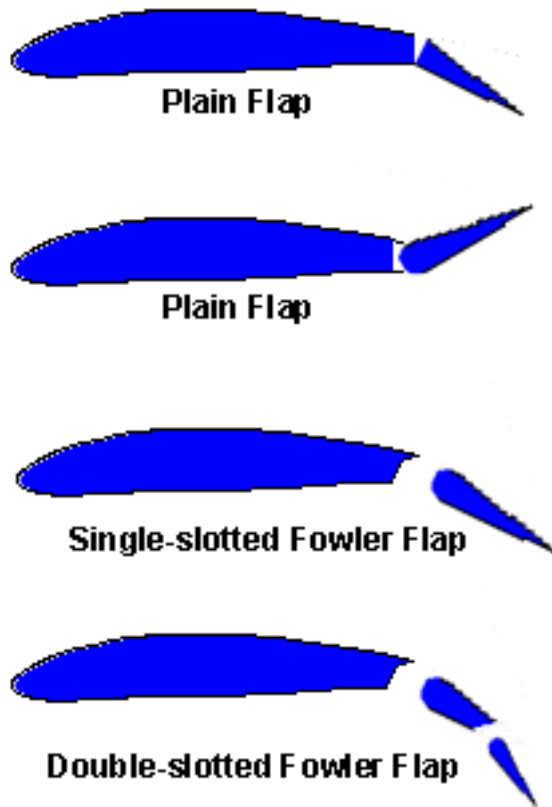


## Lift Modifying Devices

We are now in a position to understand how flaps and slats, which were mentioned earlier, work. We will also discuss wing fences, vortex generators, leading edge cuffs, wing blowers, and spoilers.

### Flaps

Figure 64 shows several different flaps types.



**Figure 64**

Plain flaps are commonly used as ailerons, elevators and rudders. You may not think of these devices as flaps, but they are. A plain flap changes the camber of a wing and thereby increases or decreases lift.

Fowler flaps are flaps that move backward as they extend thereby increase surface area at the same time they increase camber. Fowler flaps almost always are also slotted flaps.

Slotted flaps create an opening that allows air to escape from the lower surface of the wing to the upper surface. This slot is shaped so that air passing through it accelerates. The air then combines with the boundary layer on the top surface of the wing re-energizing it. You learned previously that lack of energy in the boundary layer results in stalls at high angle of attack. The slot therefore delays the stall allowing the wing to fly to a greater angle of attack.



## Slats and Slots

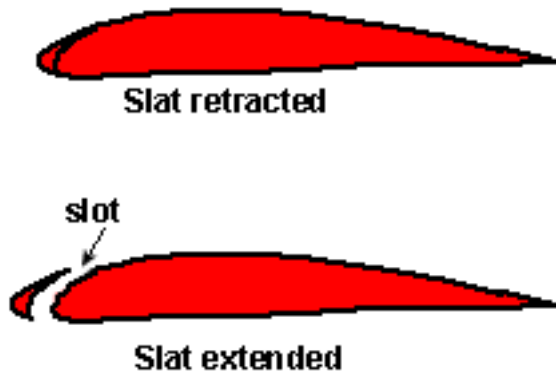


Figure 65

Figure 65 shows a device known as a slat. It extends from the leading edge of a wing creating a slot. It is the slot that does the work. The slot accelerates air from below the wing and merges it with the boundary layer above the wing thereby delaying the stall. A leading edge slot is more effective than a slot in a flap (described above.) Aeroplanes equipped with slots can fly at much greater angles of attack before stalling.

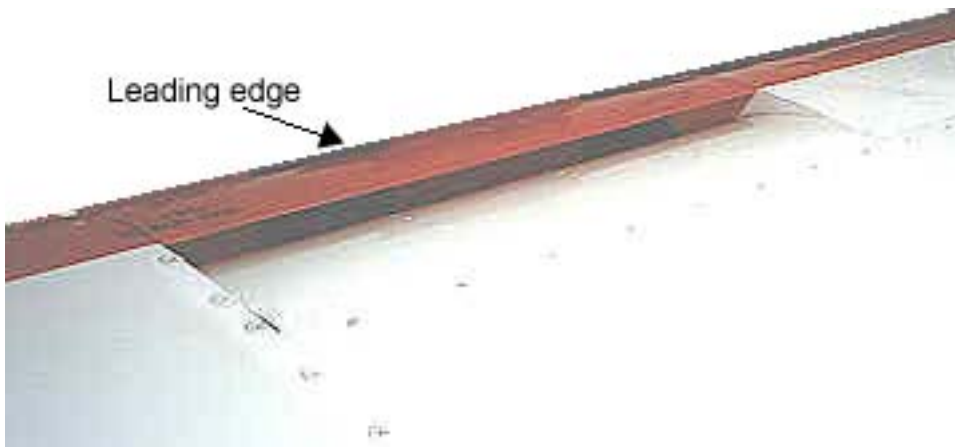
It is possible to make a slot in a wing without a moveable device, i.e. without a slat; to do so imposes a small drag penalty, but is cheaper to build. Such a slot is usually referred to as a **fixed slot**. Examples can be found on several light aircraft.



*The aeroplane pictured above has a fixed slot. I.E. the slat does not move, it remains open all the time. This has the advantage of mechanical simplicity, but it does slightly increase drag in cruise.*



*This picture shows a different style of fixed slot, photographed from below. Note that the slot is located on the outboard section of the wing, which ensures the wing-root stalls first.*



*This is the same wing slot photographed from above and looking toward the leading edge.*

## **Wing Fences**

Figure 66 shows a typical wing fence. Its purpose is to reduce span-wise movement of air flowing laterally along the wingspan. Recall our previous discussion about induced drag, in which we said that induced drag is due to this span-wise movement. A wing fence should reduce this, slightly decreasing induced drag and increasing lift. When installed, wing fences are normally at the dividing line between the flaps and ailerons and have their greatest effect when flaps are extended because the increased lift produced by extended flaps tends to induce a secondary vortex at the flaps outer tip which reduces the effectiveness of the flaps. The fence reduces this vortex making the flaps more effective.

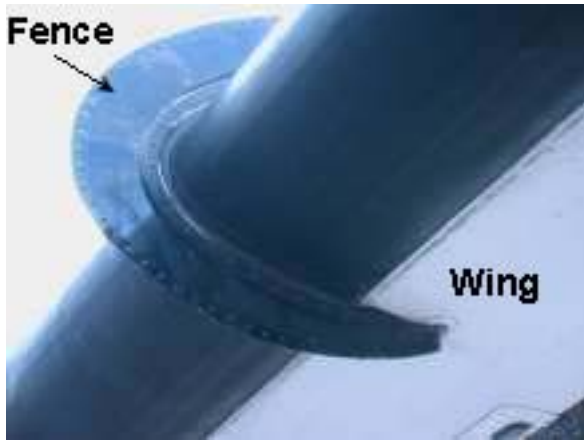


Figure 66



*This photo shows a wing fence on a Falcon jet in the BCIT hangar.*

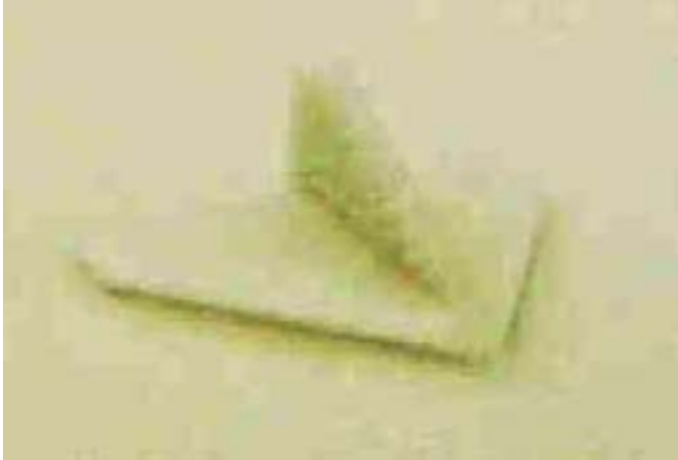
### **Vortex Generators**

We will come across vortex generators again later when we talk about supersonic flight. Their main use in aviation is to keep the boundary layer flowing through shockwaves that form near the speed of sound. On subsonic (light aircraft) they are sometimes used to energize the boundary layer to prevent flow separation. In theory this would increase the stall angle of attack. In practice however they are considerably less effective than slots. Vortex generators may also be useful on some aeroplanes to maintain the effectiveness of the rudder or elevator at high deflection angles.

If an aeroplane is poorly designed and experiencing flow separation at sharp corners then vortex generators can be used, just like dimples on a golf ball, to reduce parasite drag.



*This row of vortex generators are along the wing of a Citation X*



*This is a close-up of the vortex generators shown above.*

### **Leading Edge Cuffs**

Figure 67 shows a leading edge cuff. These cuffs are added to wings to modify the stall and spin characteristics. NASA research conducted in the 1970s indicated that spin recovery was considerably easier with cuffs installed, with minimal drag penalty. The photo is of a 1981 C-172P.

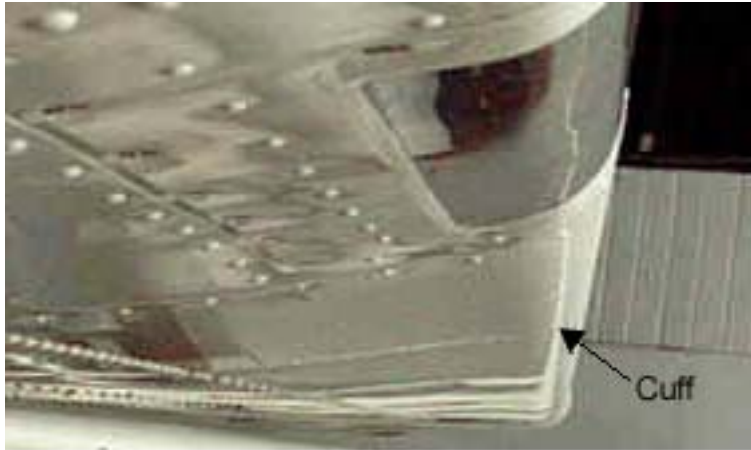


Figure 67

### Wing Blowers

Figure 68 shows schematically how a wing blower works. High pressure air is taken from the compressor section of jet or turboprop engine is injected into the boundary layer. This prevents flow separation thereby achieving the same effect as a slot.

Wing blower systems are very effective, unless the engine fails. They are however heavy and expensive. Wing blowers are not used significantly in civilian aeroplanes, to date.

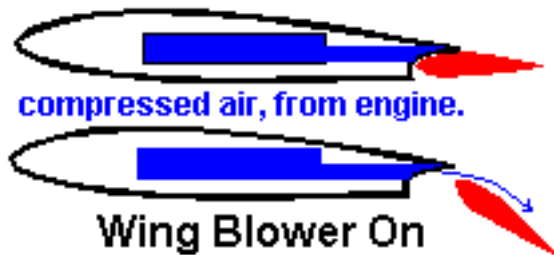


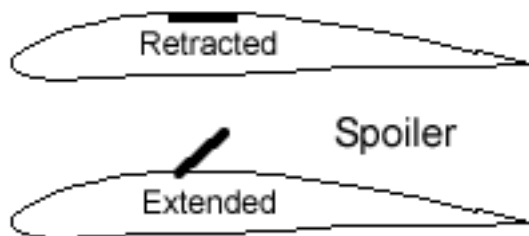
Figure 68

## Spoilers



*This photo shows a retracted spoiler typical of ones found on light aircraft.*

Figure 69 shows how the spoiler extends.



**Figure 69**

Spoilers extend into the boundary layer forcing it to separate. In effect the spoiler stalls the wing, even at low angle of attack. It might seem like an odd thing to want to do, but by extending the spoilers a pilot can simultaneously reduce lift and increase drag thereby facilitating a rapid speed reduction or descent (sometimes spoilers are called speed brakes.) To prevent yaw and roll spoilers must be deployed equally on each wing.

Many jet airliners have spoilers that are used in conjunction with ailerons to assist in roll control. In this case one spoiler extends, on the side of the up-going aileron, to add to the roll moment produced by the aileron. The advantage of this design is that smaller ailerons can be used thereby making more room for flaps, which is often desirable on jet aircraft with high wing-loading. An obvious problem with this design is that drag is created anytime the spoilers are extended, which is not desirable in cruise. Consequently it is often the case that roll-spoilers are disabled in cruise, either by the flight computer or by mechanical interconnection with the flap control.



A very few aeroplanes have been built with spoilers only for roll control, i.e. with no ailerons at all. This design has some serious drawbacks as even a slight deflection of the control wheel, as might be needed for minor load imbalance for example, results in considerable drag and loss of lift.

Spoiler float is a term that refers to the tendency of the low pressure area above the wing to “suck” a spoiler upward<sup>19</sup>, with obvious negative effects on drag and lift. To prevent spoiler float they are normally mechanically locked down in cruise. Failure of the lock-down mechanism is a serious matter that jet-pilots watch for.



*This photo shows the spoilers on an Air Cadet glider.*

Some aeroplanes have spoilers that are only approved for operation on the ground. In this case the spoilers are extended, either automatically or manually, after touchdown to slow the aeroplane and reduce lift so that wheel braking is more effective.

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<sup>19</sup> Consult the discussion of how lift is produced on page 32 for the “real reason” the spoilers rise.

## Airspeed (TAS, EAS, CAS, IAS, and Mach)

Airspeed is important to pilots for many reasons. For navigation a pilot needs to know the true airspeed (TAS) as this is the value that wind must be applied in order to calculate groundspeed and determine time enroute. Unfortunately no one has designed an instrument that measures true airspeed, although modern navigation equipment can measure groundspeed directly, bypassing the need to know TAS.

In order to fly an aeroplane safely what a pilot really needs to know is the dynamic pressure flowing past. The instrument commonly known as an airspeed indicator (ASI) is actually a DYNAMIC PRESSURE INDICATOR. Unfortunately it doesn't do a precise job of measuring dynamic pressure, for reasons about to be explained.

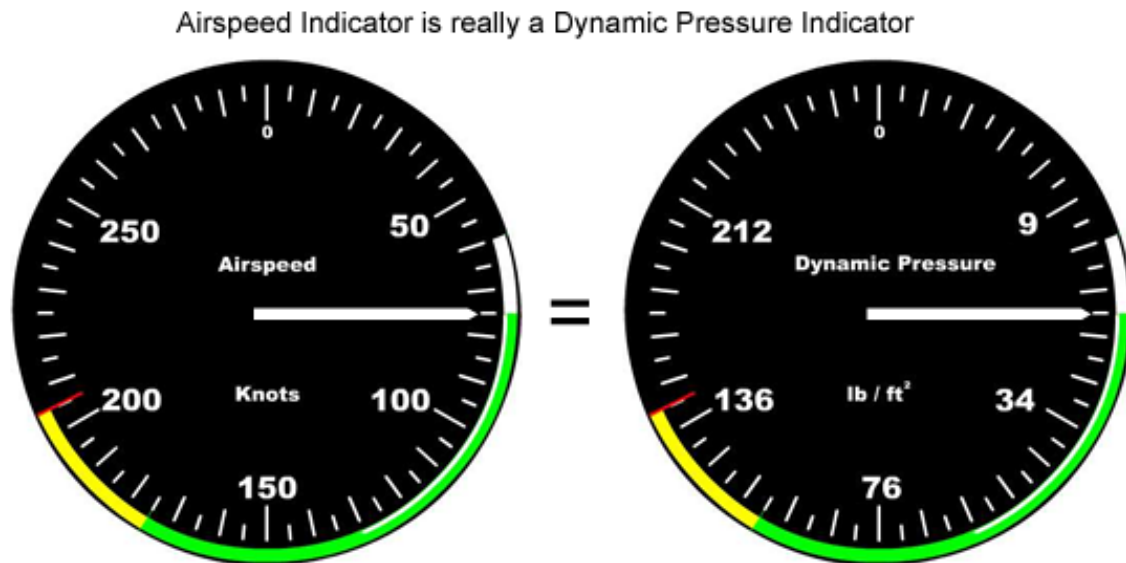


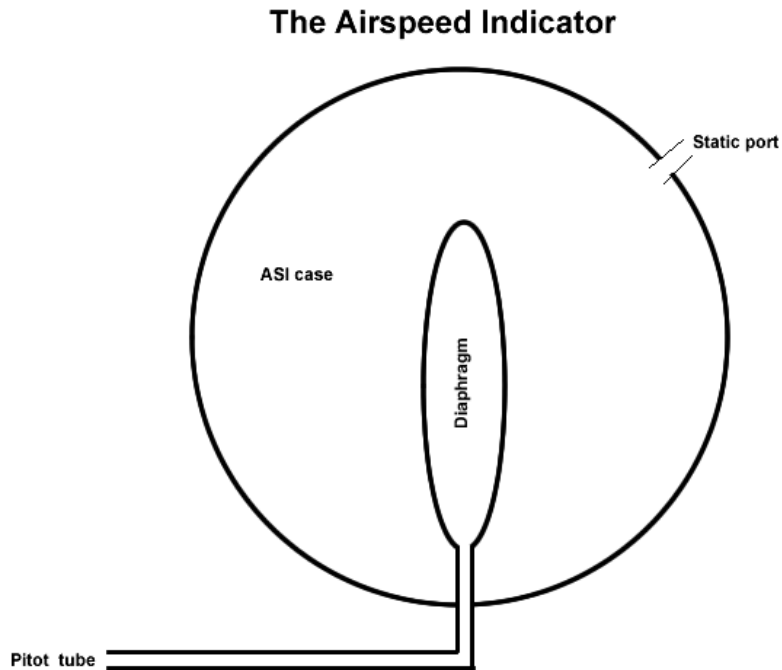
Figure 70

Keep in mind that an ASI does NOT measure TAS. The ASI in Figure 70 indicates 75 knots, but the actual speed may be much higher depending on the airplane's altitude. The actual indication is of a dynamic pressure of 19 lb/ft<sup>2</sup>, but this is equivalent to 75 knots at sea level.

### ***How an Airspeed Indicator Works***

Dynamic pressure,  $q = 1.426 \rho V^2$  is what an ASI is designed to measure. The importance of dynamic pressure should be obvious since it appears in both the lift and drag equations.





**Figure 71**

As shown in Figure 71, a Pitot tube is oriented directly into the oncoming relative wind to measure “ram air” which is routed through a tube to a diaphragm in the ASI. The more ram air there is the more the diaphragm expands. Static air in the case escapes through a static port. Without a static port the expanding diaphragm would compress the air in the case and be unable to expand.

The total pressure inside the diaphragm equals the sum of static and dynamic pressure; this pressure expands the diaphragm. Since air in the case is at static pressure, and this squeezes the diaphragm, it seems the ASI should successfully measure dynamic pressure (i.e. the size of the diaphragm should be proportional to dynamic pressure.)

Let  $q$  represent dynamic pressure and consequently the size of the diaphragm in the ASI, then:

$$q = 1.426 \rho V^2$$

$$V = \sqrt{(q / 1.426\rho)}$$

The ASI displays airspeed ( $V$ ) based on the above equation. The value of  $\rho$  used in calibrating the instrument is 0.002377, i.e. sea level standard air density. If the dynamic pressure is perfectly measured an ASI can provide an accurate representation of dynamic pressure, in the form of *sea-level equivalent speed*. But unfortunately there are two errors that prevent completely accurate measurement of dynamic pressure.

## Calibration Error – Position Error

The static port is difficult to locate on a moving aeroplane. If it captures any dynamic pressure the ASI will under read (be sure to reason out why) On the other hand the actual static pressure along the side of a moving aeroplane is often lower than atmospheric pressure, and if the static port experiences this it will cause the ASI to over read (can you see why?). The aircraft designer must endeavor to place the static port so that it experiences actual atmospheric static pressure. The chosen point is usually somewhere on the side of the fuselage. Unfortunately static pressure at a given point on the fuselage tends to change with angle of attack making it nearly impossible to choose a spot that is ideal at all flight speeds.

When angle of attack changes ram air enters the Pitot tube at a slight angle which causes a further calibration error.

The error due to static-port location is called *position error*.

The total error due to both dynamic and static measuring errors is call *calibration error*.

Calibration error differs for each type of aeroplane, and usually varies with angle of attack. It must be looked up in the POH. For the C-172P you can find airspeed calibration corrections on page 5-8 of your POH.

## Compression Error

The second source of error when measuring dynamic pressure with an ASI is called compression error.

Based on what was said above it seems logical that diaphragm expansion is proportional to dynamic pressure, but that is only the case if air density inside the diaphragm is the same as air density in the case, which it is not.

Ram air entering the Pitot tube is compressed when it is brought to a stop in the diaphragm. Therefore air inside the diaphragm is denser than that in the case.

If this density difference was always the same it could be allowed for in the design of the instrument. Unfortunately thinner air at high altitude is more easily compressed than the thick air at sea level. A good ASI is correctly adjusted for compression at sea level but over reads at high altitude – I.E it indicates more dynamic pressure than really exists.

## ICE-T

To get from indicated airspeed to true airspeed you must progress through the sequence:

1. Indicated airspeed (IAS)
2. Calibrated airspeed (CAS)
3. Equivalent airspeed (EAS)
4. True airspeed (TAS)

This can be remembered by the pneumonic ICE-T.

To convert IAS to CAS apply the calibration correction

To covert from CAS to EAS apply the compression correction

EAS accurately reflects the dynamic pressure and as such is the desired value for piloting (but not navigating.) EAS is frequently not much different than IAS, which is fortunate since that means pilots can fly safely by referring to the IAS most of the time. But EAS is the proper reference and therefore pilots need to know under which circumstances IAS is substantially different than EAS. In other words pilots need to know what circumstances cause a lot of calibration and compression error.

### **Calibration Error**

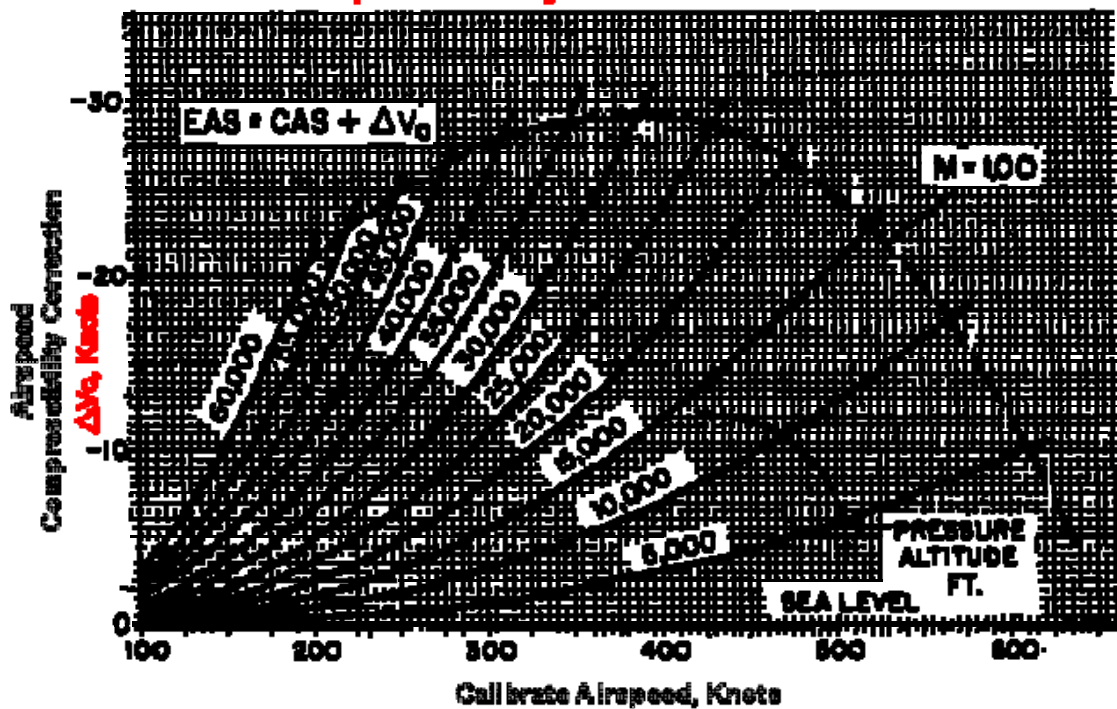
Calibration error is usually greatest at the extremes of the operating envelope, i.e. at very low and very high indicated airspeeds. In light aircraft this often means that the ASI is unreliable when in slow flight (but usually quite reliable in cruise.) In a jet airplane that flies over a much wider range of airspeeds the ASI may have significant calibration errors at both low and high speeds.

Calibration error is applied to IAS to get CAS. CAS represents what a perfect ASI should read, i.e. one with no calibration errors. If compression error is small CAS can safely be used as a flight reference. This is the case for all light aircraft, but in a jet pilots must also consider compression error.

### **Compression Error**

A compression correction factor must be applied to the CAS to get EAS.

## Compressibility Correction Chart



The chart above can be used to convert CAS to EAS. Note that EAS is always less than CAS.

At 200 KCAS and 10,000' compression error is about 2 knots. This is insignificant and can be ignored. At 200 KCAS and 20,000' compression error is about 5 knots, which is enough to require consideration.

Aeroplanes that travel faster than 200 knots **and** higher than 20,000' require compression allowance. Slower, low flying aeroplanes do not.

What is the EAS of an aeroplane flying at 350 KCAS at 45,000'? \_\_\_\_\_

### Density Error

EAS is the speed that determines aerodynamic performance i.e. it represents the dynamic pressure – in units of sea level equivalent airspeed<sup>20</sup>. For navigation it is necessary to convert this to the actual velocity i.e. true airspeed (TAS.) To do this air density must be allowed for

By definition:

$$V_{EAS} = \sqrt{(q / 1.426\rho_{SL})} \quad \text{[use sea level air density to get EAS]}$$

<sup>20</sup> An ASI could easily be calibrated to read pressure in psi or pounds per square foot, but it is easier for pilots to grasp the unit of sea level equivalent airspeed. Just keep in mind that when you fly at any altitude above sea level your true airspeed is more than the sea level equivalent.

$$V_{TAS} = \sqrt{(q / 1.426\rho_{actual})} \quad [\text{use actual air density to get TAS}]$$

Therefore the relationship between TAS and EAS is given by:

$$TAS = EAS \times \sqrt{(\rho_{SL} / \rho_{actual})}$$

Table 2 gives a few sample values of  $\sqrt{\rho_{SL} / \rho_{actual}}$ . You can calculate more by referring to table 1.

Altitude	$\sqrt{\rho_{SL} / \rho_{actual}}$
Sea level	1.00
5000	1.08
10,000	1.16
15,000	1.26
20,000	1.37
25,000	1.49
30,000	1.63
40,000	2.02
50,000	2.56

**Table 2**

Table 2 shows that at sea level TAS = EAS, at 5000 feet TAS is 8% more than EAS, at 40,000 feet TAS is twice EAS.

When you use your CR3 flight computer to get TAS it allows for compression error automatically (if you use the professional method.) Thus, with the CR3 you set CAS opposite pressure altitude and then read TAS after allowing for indicated air temperature. EAS is never displayed to you even though it is allowed for in this process. If you apply calibration correction to CAS to obtain EAS you must not use the CR3 professional method to get TAS as you will effectively be allowing for compression twice.

### Mach Meters

A Mach meter is an airspeed indicator that displays speed in units of Mach. There are two types of mach meter known as type A and type B.

Mach is defined as  $TAS/a$  [a is the local speed of sound]

The speed of sound (a) depends on air temperature as explained earlier in this text. You might think that it would be necessary for a Mach meter to have a temperature sensor, but actually it is not. The reason is that air density also depends on air temperature. You can see in the following mathematical development which uses the equations for TAS and speed of sound presented earlier to develop a formula for Mach number that depends only on EAS and  $P_s$ :

$$M = \frac{TAS}{a} = \frac{EAS \sqrt{\frac{\rho_{SL}}{\rho_{actual}}}}{38.98 \sqrt{T}}$$

$$\rho = \frac{P_s}{3089.54 T}$$

$$M = \frac{EAS}{14.4 \sqrt{P_s}}$$

When the second equation is substituted into the first the temperature cancels out leaving Mach proportional only to EAS and the inverse of the square root of static air pressure. In short, no temperature input is needed.

The early Mach meters that were used until the 1980s, and are still used in some business jets today, were type A, designed based on the above formula. They work quite well but must approximate compression (CAS to EAS correction) based on altitude. Modern Type B Mach meters use an air data computer (ADC.) An ADC has a temperature sensor and is able to correctly allow for temperature rise. It can therefore accurately calculate the speed of sound and the true airspeed, and therefore the Mach number. Type B Mach meters usually display Mach number to three decimal places and are very accurate. Type A Mach meters are less accurate but adequate for safe flight operations.

Note: On your CR3 computer you should notice that if you set CAS opposite pressure altitude you get a specific mach number. This value does not depend on the indicated air temperature. Indeed the proper procedure to get TAS with a CR3 is to take the Mach number and convert it to TAS, which does require you to allow for air temperature.

## Flight for Range and Endurance – Jet aeroplanes

It is assumed here that you have covered the theory of how a jet engine works elsewhere. For our purposes you must know that for a jet fuel flow (FF) is proportional to thrust. The equation is:

$$FF = tsfc \times T \quad [FF \text{ is fuel-flow in lb/hr. } T \text{ is thrust in pounds.}]$$

Consequently:

$$T = FF / tsfc$$

TSFC is called the *thrust specific fuel consumption*. It specifies how much thrust is produced for every pound of fuel consumed. Values of tsfc vary from engine to engine and are improving with each new generation of engine. Typical tsfc values range from as low as 0.8 to as high as 1.3 lb/lb/hr, and will likely increase in the future.

What factors affect tsfc? There are two that pilots can control:

1. High engine rpm increases tsfc
2. Low air temperature increases tsfc

In other words throttling back to low engine rpm spoils the efficiency of a jet engine. Also flying at high altitude where the air is cold is a good idea.

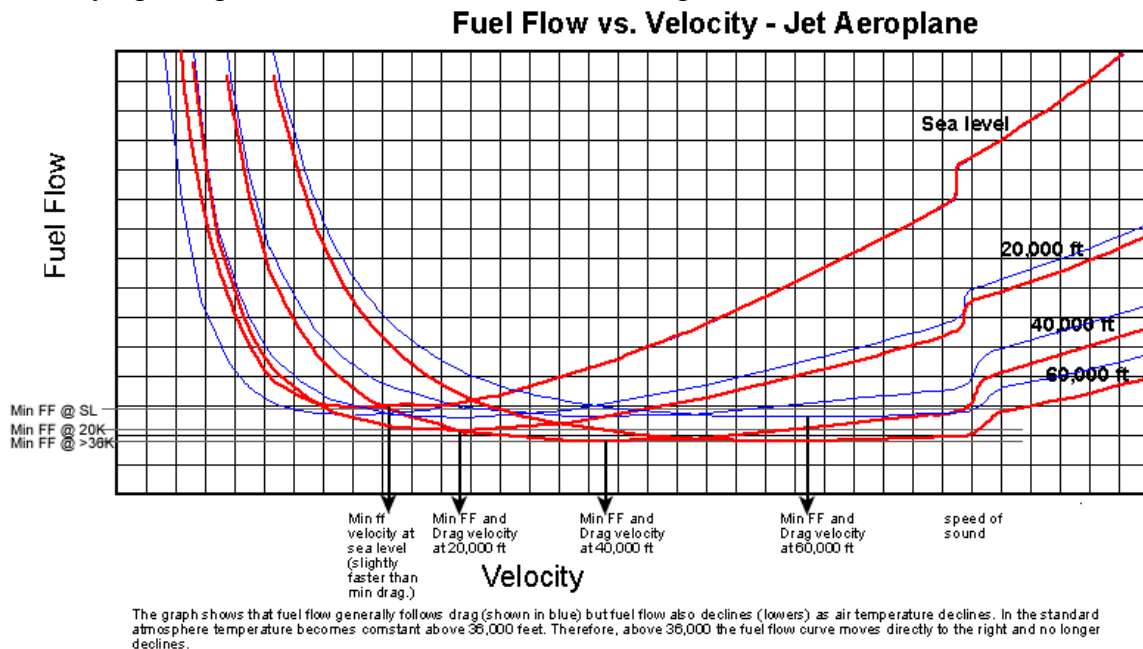


Figure 72

Figure 72 shows a graph of Fuel Flow vs. Velocity for a jet aeroplane in straight and level flight at several altitudes. The graph was created by setting thrust equal to drag, which is correct for *level flight* and then dividing thrust by tsfc to get fuel flow (see definition of tsfc above.) The shape of the curve is *almost* identical to the total drag curve (page 85,) note the following points:

- The FF-curves shift to the right with altitude because reduced air density decreases parasite drag but increases induced drag.
- The lowest fuel-flow for level flight decreases up to 36,000 feet because reducing air temperature with altitude improves tsfc. Above 36,000 feet (in the stratosphere) temperature is constant so tsfc becomes constant and the base of the FF-curve shifts right without further lowering.
- At low altitude the left side of the FF-curve is “distorted” upwards (i.e. to higher fuel flow) because low engine rpm would be required to fly at such speeds.

We define **flight for endurance** as flying with the lowest possible FF. To do that means to fly at the bottom of the FF-curve. What speed and altitude should we operate at for maximum endurance?

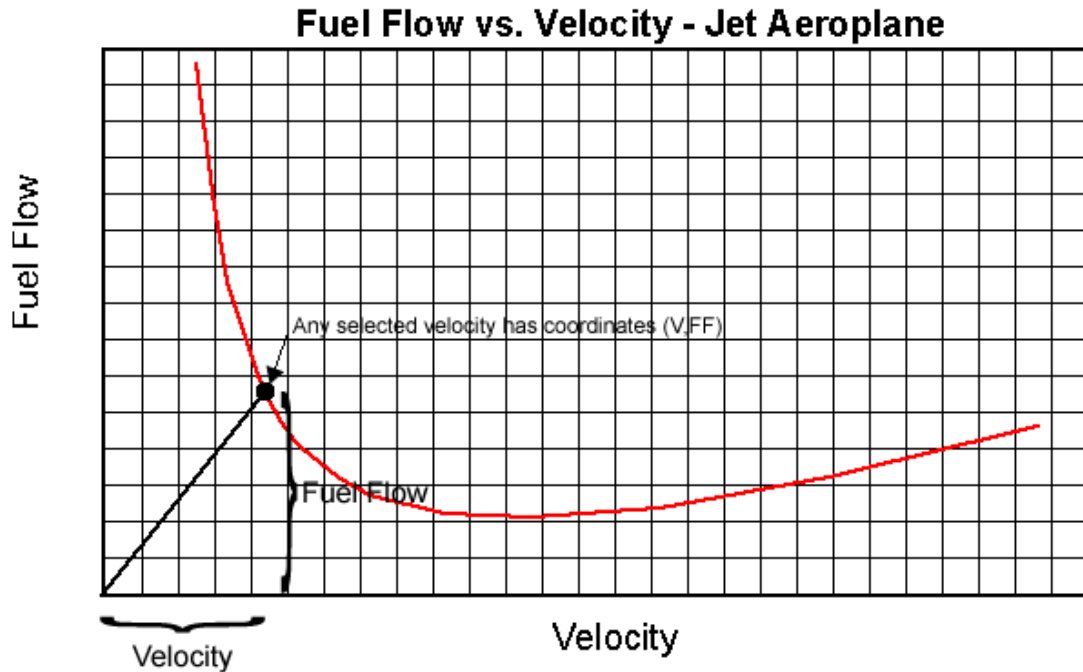
We should fly at *36,000 feet or above* and operate at the true airspeed corresponding to the bottom of the fuel flow curve. It should be clear that this speed will correspond to flight at the minimum drag speed<sup>21</sup> i.e. at  $L/D_{\max}$  angle of attack.

We define **flight for maximum range** as flight with the maximum ratio of Groundspeed/Fuel flow. In car terminology we would say “maximum miles per gallon.” Since the x-axis of the graph represents  $V$  and the y-axis represents FF the x/y coordinate of each point on the curve represents its **specific range** ( $SR = \text{groundspeed}/FF$ ) if there is *no wind* (i.e. groundspeed =  $V$ .) Initially we will deal with the no wind situation. The zero-wind SR concept is demonstrated in Figure 73. What point on the curve has the largest zero-wind SR?

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<sup>21</sup> At or near sea level it might be necessary to fly slightly faster than the minimum drag speed in order to avoid operating the engines at an inefficient, low, rpm.

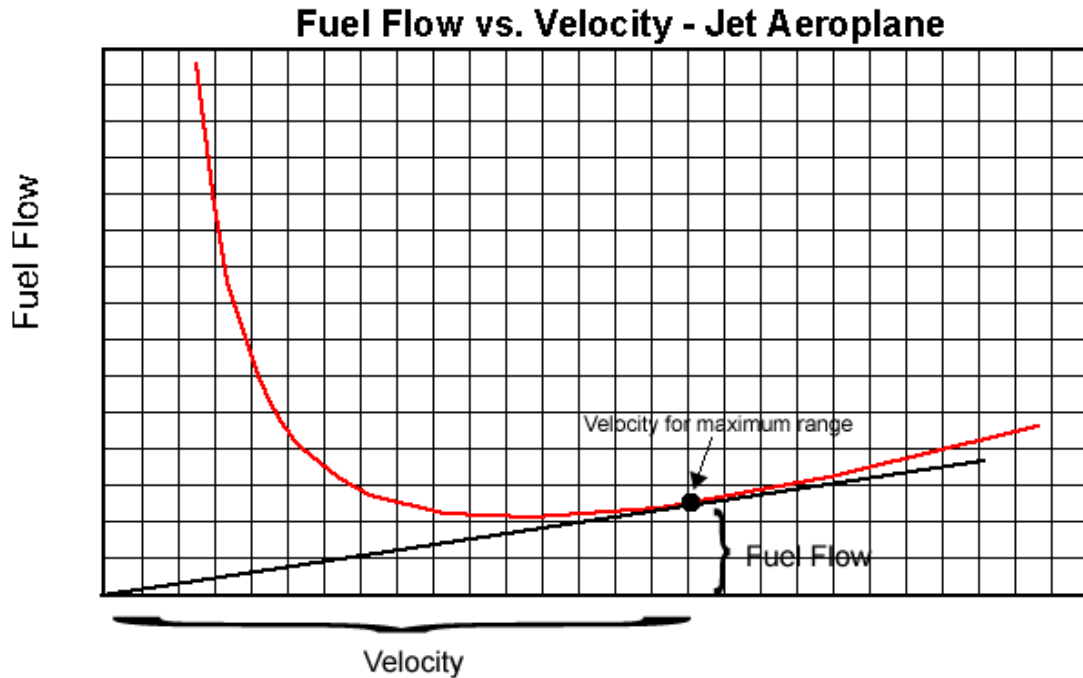




The above graph shows that specific range (SR) is calculated for any chosen velocity by taking the points "x-coordinate" the velocity and dividing by the points "y-coordinate" the fuel flow.

**Figure 73**

Draw a line from the graph's origin tangent to the FF-curve, as shown in Figure 74. The point where the tangent line touches the curve has the maximum ratio of  $V/FF$ , or you might find it easier to say that it has the minimum ratio of  $FF/V$ . This would be the speed for maximum SR in zero wind. This speed corresponds to flying at the angle of attack that produces  $\sqrt{L/D_{\max}}$ .

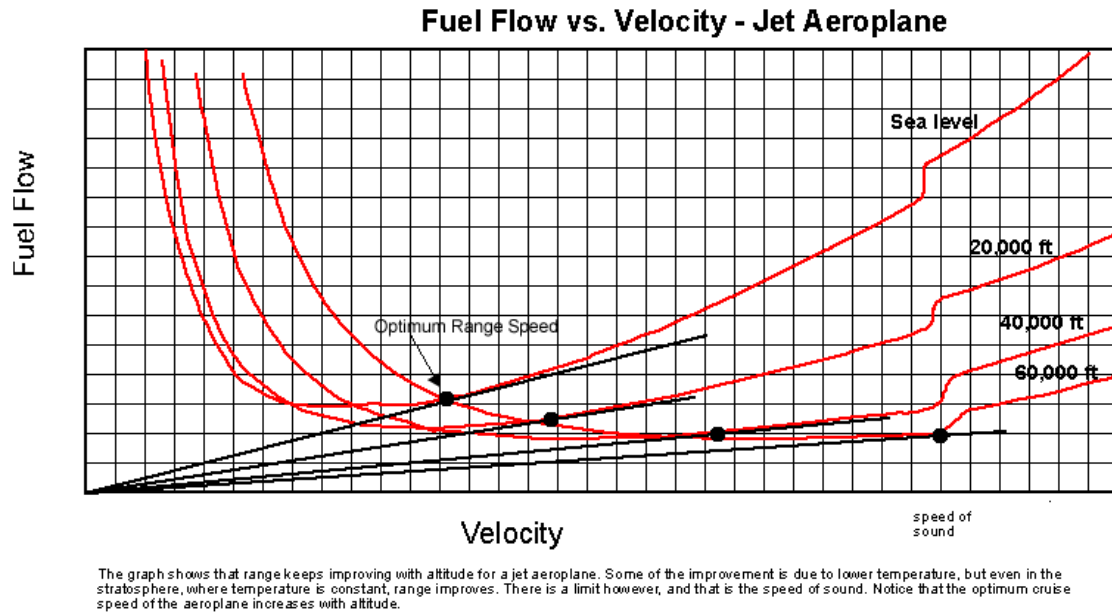


Assuming no wind (i.e. groundspeed = velocity) maximum range is always at the speed shown here.

**Figure 74**

It is important to see that the speed for maximum range must always be faster than for maximum endurance (which is at the bottom of the FF curve.) How does altitude affect range of the jet aeroplane?

Figure 75 shows a series of tangent lines drawn to FF-curves for several altitudes. It should be clear that SR (the ratio of  $V/FF$ ) improves with altitude, unlike endurance, which stops increasing at 36,000 feet. From the graph you can see that SR keeps improving with altitude until the speed of sound intervenes. Therefore in zero wind, a jet aeroplane should fly as high as it is able without encountering drag rise associated with the sound barrier. We will deal with the transonic and supersonic flight later.



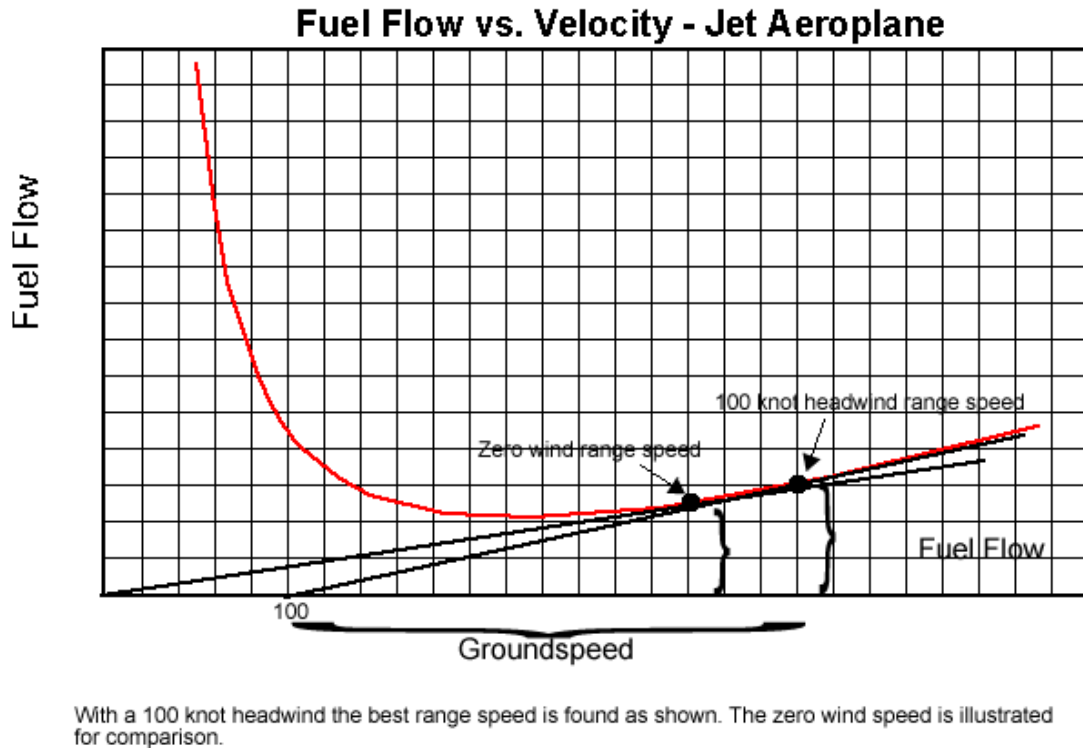
**Figure 75**

*It is worth noting that jet engines produce less thrust with altitude so there comes an altitude at which the aeroplane does not have enough thrust to climb higher.*

How does wind affect range and endurance for the jet aeroplane?

*Wind has no effect on endurance* because groundspeed is immaterial. The aeroplane may be simply holding (circling) when flying for endurance. Maximum endurance is always found at the bottom of the FF-curve and the same endurance can be achieved at any altitude from 36,000 feet to the top of the stratosphere in the standard atmosphere. In the real atmosphere endurance improves if the temperature is colder than normal and decreases if it is warmer. Maximum endurance in a jet always requires flying at the angle of attack for maximum lift to drag ratio ( $L/D_{max}$ .)

Wind has a substantial effect on SR. **When evaluating SR we should use the groundspeed, not the TAS.** I.E.  $SR = \text{groundspeed}/FF$  not  $TAS/FF$ . Graphically we can adjust the tangent line technique shown previously to allow for wind. Figure 76 shows how. A headwind of 100 knots means that  $\text{groundspeed} = TAS - 100$ , so the tangent line is simply drawn starting at 100, so that it is 100 knots shorter than the TAS. The important thing to notice is that it now touches the curve at a higher speed. Consequently, when flying into strong headwinds airspeed must be increased if optimum range is to be achieved. Note also that the ratio of  $V/FF$  is clearly less with headwind, so as common sense would indicate, an aeroplane achieves less range with a headwind.

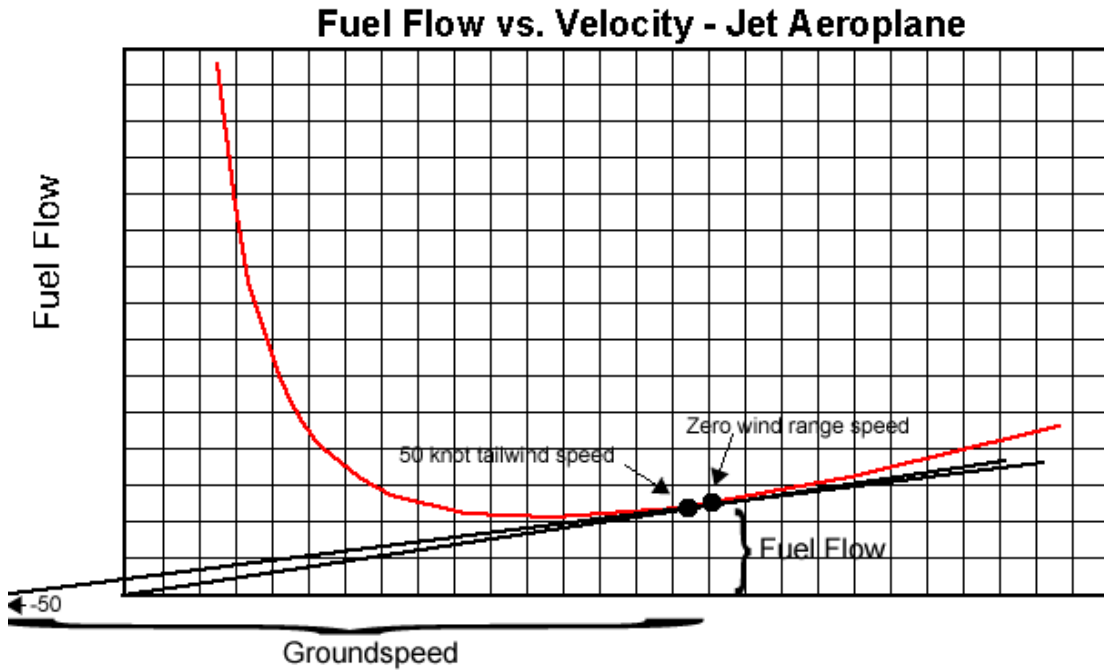


**Figure 76**

A rule of thumb that many pilots use is to increase airspeed by half the headwind. Try plotting a few sample lines on the provided charts and you will see that the rule is a reasonable one.

A similar process can be used for analyzing tailwinds. Figure 77 shows a 50-knot tailwind. An aeroplane should be slowed down with a tailwind, although not as much as it would be speeded up for a 50-knot headwind. So, while you might slow down with a strong tailwind the required speed reduction is less than half the wind speed (many pilots don't bother.)

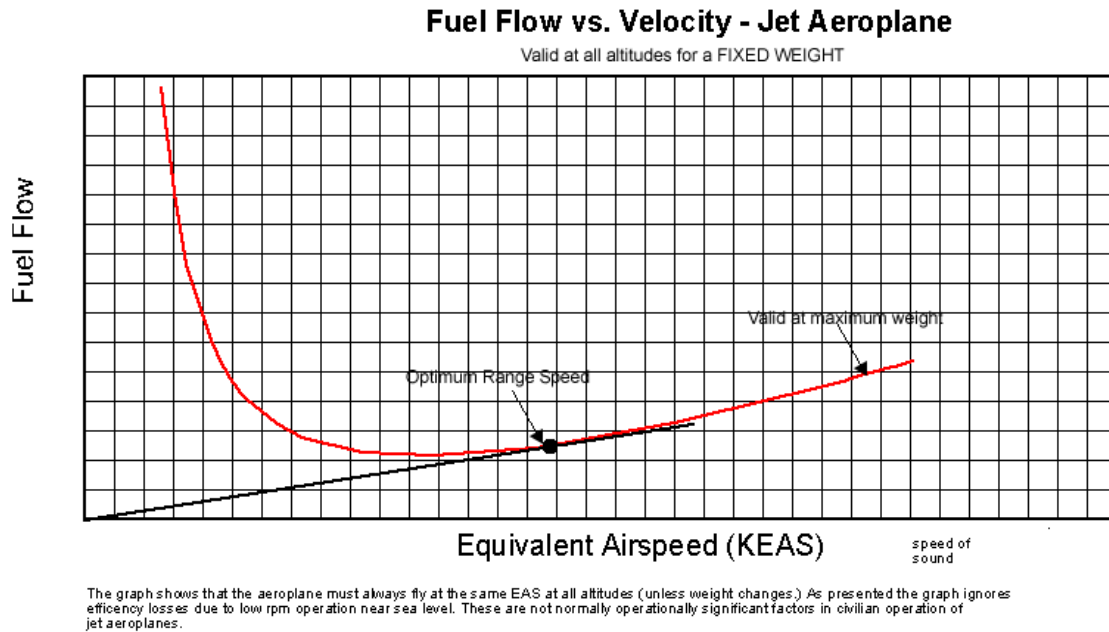
We saw earlier that in zero wind maximum range occurs at the angle of attack at which the ratio of square root of lift over drag is maximized ( $\sqrt{L/D_{\max}}$ ). With a headwind the pilot must fly with less angle of attack, and with a tailwind we should fly with a greater angle of attack to achieve maximum range.



With a 100 knot headwind the best range speed is found as shown. The zero wind speed is illustrated for comparison.

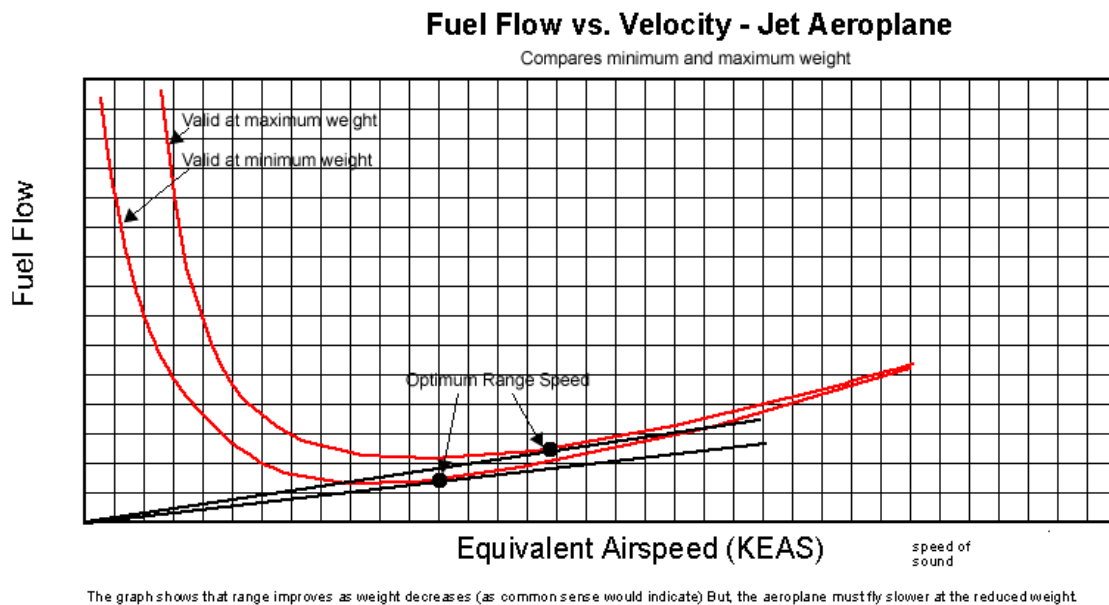
**Figure 77**

Now consider Figure 78, this figure shows the Fuel flow vs. Velocity graph for an aeroplane at maximum weight. How will this graph change as the aeroplane consumes fuel and becomes lighter?



**Figure 78**

How does weight affect the range of a jet aeroplane? We know that as weight decreases parasite drag does not change but induced drag decreases. It is important to be able to visualize how this changes the shape of the FF-curve?



**Figure 79**

Figure 79 shows that the reduction in induced drag as fuel is burned results in the left side of the FF-curve dropping. As a result range improves (notice that the ratio  $V/FF$  is

greater) but the aeroplane must fly at a slower equivalent airspeed. Since the reduction in EAS corresponds to the reduction in lift required (because weight has decreased) the angle of attack ( $C_L$ ) that the wing must fly at remains constant. This leads to some quite simple “rules” for operating jet aeroplanes efficiently.

- To fly a jet aeroplane for maximum range it must be flown at a specific angle of attack. (corresponding to  $\sqrt{L/D_{\max}}$  in zero wind.)
- This angle of attack applies regardless of weight (but does vary with wind.)
- EAS must decrease as weight decreases.

### **Cruise Control**

Even though EAS and consequently CAS and IAS must decrease as a flight progresses it is possible to maintain a constant TAS, if the aeroplane climbs to successively higher altitudes as weight decreases. This procedure, commonly called “step-climbing”, is in fact the standard method of operating large jet aeroplanes that undergo substantial changes in weight during the course of a flight.

In summary, the procedure is to establish flight at the optimum angle of attack and at the altitude at which the desired TAS or Mach number is reached. As fuel is burned EAS decreases (to keep the angle of attack constant) but by climbing to a higher altitude the desired TAS and Mach number are maintained.

### ***Flight for Range and Endurance – Propeller aeroplane***

Propeller aeroplanes can have piston or turbine engines. When a turbine engine is installed it is called a turbo-prop. Both types of engines have in common that fuel flow is proportional to power, not thrust. We write:

$$FF = \text{sfc} \times HP$$

SFC is called the *specific fuel consumption*. It specifies how much power is produced for every pound of fuel consumed. Values of sfc vary from engine to engine but are typically in the range 0.5 to 0.8 HP/lb/hr.

What factors affect sfc? The answer is slightly different for piston and turbo-prop engines. For turboprops the same factors that affect jet engines apply:

1. High engine rpm increases sfc
2. Low air temperature increases sfc

For piston engines rpm and air temperature have negligible effects on sfc. The main factors are:

1. Proficiency of mixture leaning by the pilot
2. Open throttle is more efficient
3. Using fuel to cool the engine reduces efficiency

The first factor, quality of leaning, is pretty obvious; future aeroplanes will likely have automated leaning systems that take care of the chore for the pilot, but in a manual leaning system no amount of theoretical knowledge will achieve maximum range or endurance if the pilot does not lean the mixture as per the pilot-operating handbook.

Throttle position is a relatively minor factor but the underlying theory goes like this:

1. When the throttle is partially closed it reduces the manifold pressure
2. Since the engine must “suck” air in against the vacuum efficiency goes down.

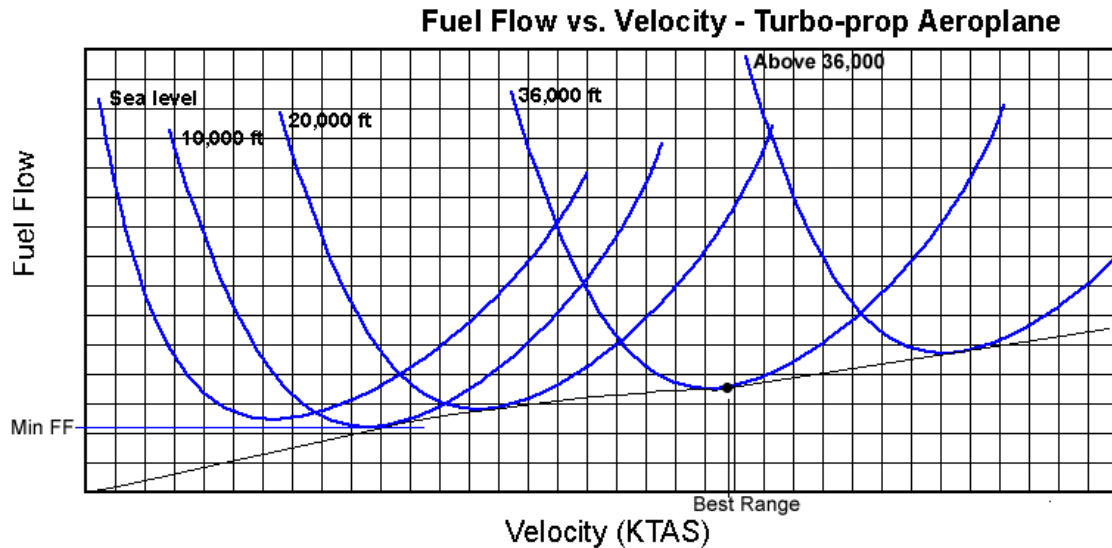
Consequently, with a piston engine it is best to climb to an altitude at which full throttle can be used. For normally aspirated engines that is usually 5000 to 10,000 feet asl. As you know if you have ever operated a piston engine with a constant speed propeller you can set various combinations of manifold pressure and rpm to get any desired power setting. However the pilot operating handbook will provide limitations on the maximum manifold pressure that can be used for a given rpm, therefore to make your engine as efficient as possible climb to an altitude at which full available manifold pressure can be used. (Note that we will shortly see that wind is a much more important factor in selecting a cruising altitude.)

When turbo-chargers are fitted to piston engines they heat the air entering the engine. If the air is heated too much extra fuel must be added to prevent detonation. This reduces efficiency. The practical result is that turbo-charged engines have some altitude above which their efficiency begins to deteriorate.

Earlier you learned how to calculate THP.  $THP = T \times V / 325.6$ . Assuming that thrust equals drag we can apply the sfc formula above to create a Fuel Flow vs. Velocity in level flight graph.

Figure 80 shows a graph of Fuel Flow vs. Velocity for a **turbo-prop** aeroplane at various altitudes. Note the tangent line from the origin to each curve. The curves shift up along a distorted tangent line. The line is distorted below 36,000 feet due to declining air temperature. Above 36,000 the fuel flow curves “slide” along a straight line as shown. In addition the sea-level curve is above the tangent line, as shown in Figure 80, because engine rpm is lower than optimum. What altitude would you fly at for maximum endurance or maximum range?





Power and therefore fuel flow requirements increase with altitude for the turbo-prop aeroplane. Consequently the fuel-flow curve slide up along the tangent line drawn from the origin. But the tangent line is distorted, as shown, below 36,000 feet due to decreasing air temperature with altitude. Consequently specific range improves slightly up to 36,000 feet. Above that altitude SR is constant.

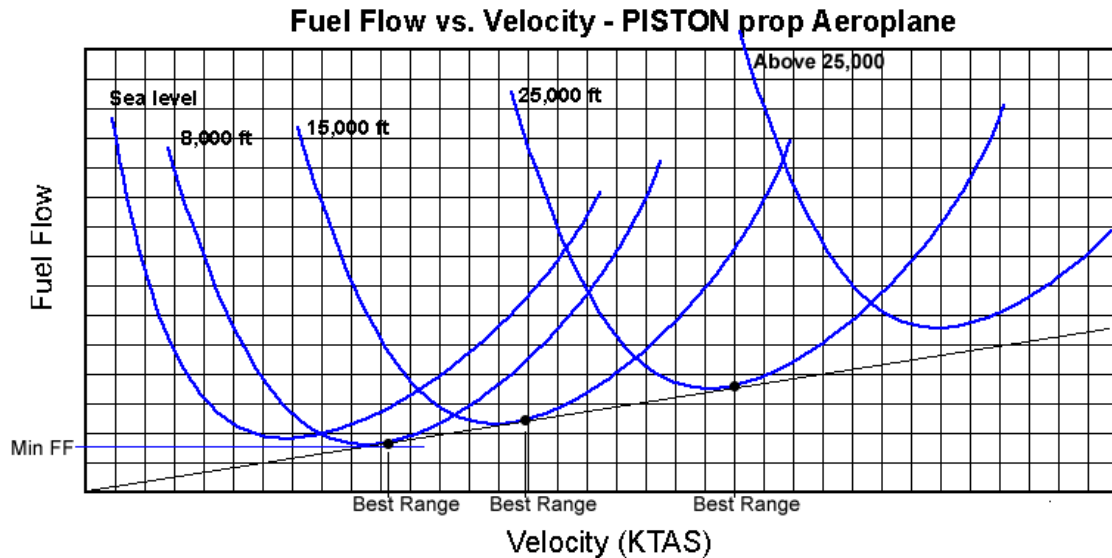
**Figure 80**

For the turboprop aeroplane maximum endurance clearly occurs close to sea level. Most turbo-props achieve maximum endurance between 5,000 and 10,000 feet; just high enough to get the engine rpm up to optimum.

For the turboprop aeroplane maximum range occurs at 36,000 feet. But, as you can see, the penalty for flying lower is not nearly as great as for the jet aeroplane examined earlier.

The bottom of the FF curve is NOT  $L/D_{\max}$  for the propeller aeroplane.  $L/D_{\max}$  actually occurs at the point where the tangent line touches the curve. In other words the maximum range speed for a propeller aeroplane occurs at the same angle of attack (and speed) that a comparable jet aeroplane would fly for maximum endurance.

To be realistic, it is *hardly ever desired to fly for maximum range* in a turbo-prop. A much more dominant concern in real world flying is speed, because as someone once said “time is money.” You can clearly see that maximum-range TAS increases with altitude, i.e. low altitude flying requires flying very slowly which is usually not practical. Most pilots will choose to fly at faster (at high power setting.) Therefore *the zero-wind cruise efficiency of a turboprop aeroplane improves with altitude*. This amounts to saying that for a given amount of power (usually represented by engine-torque) you fly faster at a higher altitude – so you should fly high. The precise rule should be to fly at the altitude at which you get the maximum ground speed (i.e. take wind into consideration) Wind speed often increases with altitude, but so does TAS, therefore with a tailwind fly as high as the aeroplane is able; but if there is an increasing headwind with altitude determine whether the increased true airspeed and decreasing fuel flow due to temperature drop offsets the increased headwind and choose the altitude with the maximum efficiency.



**Figure 81**

Figure 81 shows a graph of Fuel Flow vs. Velocity for a turbo-charged-piston-engine aeroplane. Essentially the FF-curves "slide" along the tangent line. A very slight deficiency occurs below 8000 feet due to "throttling" and above some altitude, arbitrarily set at 25,000 feet in this example, due to cooling requirements.

A piston engine aeroplane achieves maximum endurance very close to sea level. The sample aeroplane in the figure achieves maximum endurance at 8000 feet.

**Maximum range is largely unaffected by altitude** for the piston aeroplane. But what about practical operational range? We will discuss that shortly.

How does wind affect maximum endurance and maximum range for piston aeroplanes?

As with a jet, wind has no effect on endurance.

Wind affects range for the piston aeroplane. The same graphical analysis demonstrated in Figure 76 could be applied to the piston aeroplane. This analysis will give the correct speed to fly for maximum range. However this speed is usually slower than the speed most pilots desire to fly. Therefore the more appropriate procedure is the same as with the turbo-prop aeroplane discussed above. Pilots should choose their desired power setting and fuel flow and fly at the altitude that gives the maximum groundspeed. In zero wind or with a tailwind that means flying as high as possible (other non-aerodynamic factors being ignored here.) With a headwind the pilot should determine the altitude that gives the maximum groundspeed. Of course you must always consider the fuel used to climb to altitude, which is ignored in the above analysis. Thus it is only worth climbing to a high altitude if the trip is of considerable length.

Weight affects the range of a propeller aeroplane. Just as with a jet aeroplane maximum range is achieved at a specific angle of attack ( $L/D_{\max}$  in zero wind.) In theory pilots should slow down as fuel is consumed. However, propeller pilots hardly ever fly at  $L/D_{\max}$  anyway (because it is too slow to be practical) and so don't slow down as fuel is consumed. While this is fine and appropriate in most cases it is worth knowing that in theory you should slow down as fuel is consumed if you ever get caught in a situation where you are trying to stretch your fuel to maximum range.

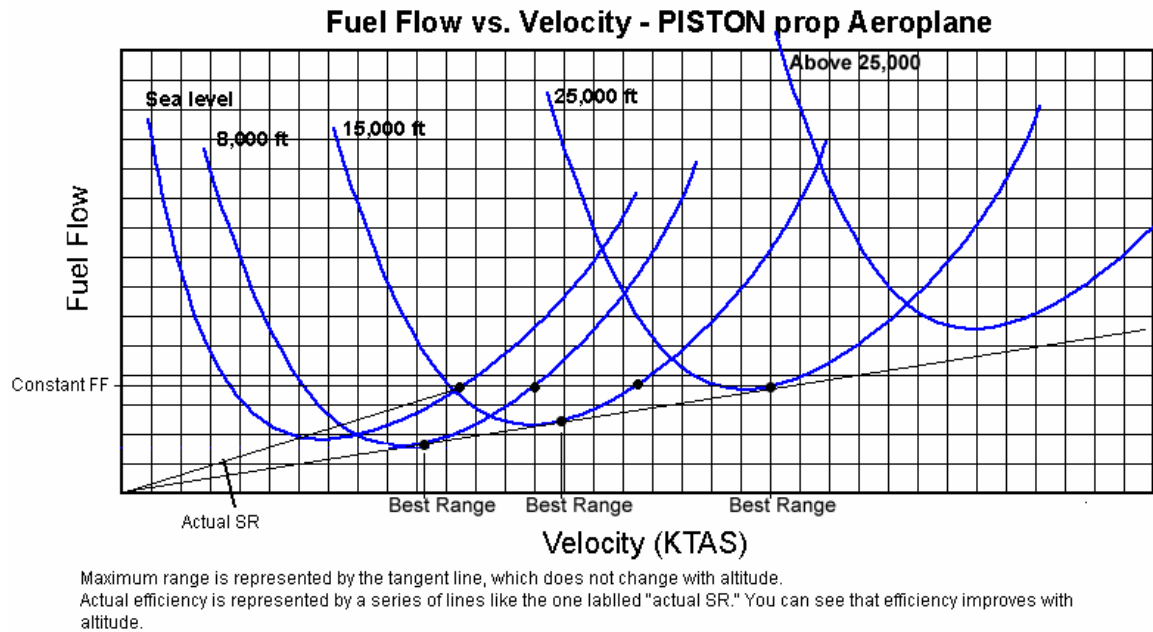
If you fly the way most pilots do you probably let your TAS increase when the airplane is lighter (reduces induced drag) and consequently range improves slightly. Most of your efficiency gains are due to climbing faster at a reduced weight. While this factor is very operationally important we will not delve into it further here.

### Practical Flight for Range in a Piston aeroplane

Take note as you examine Figure 81 that the power setting for **maximum** range increases with altitude. I.E. at sea level the aeroplane may require 20% power for maximum range and that might rise to 40% at the aeroplane's service ceiling. The numbers just mentioned are examples only, but for most aeroplanes the power setting required for maximum range is quite low; well below the value most pilots use in real world flying (remember that fact if you ever get caught in an emergency situation.)

Reexamine Figure 81 and imagine you are flying at sea level but using a fuel flow typical of actual flight operations. This situation is shown in Figure 82. You would be flying "too fast" for optimum range at sea level. If you climbed to a higher altitude but maintained the same fuel flow your range would improve. Imagine a series of tangent lines drawn from the origin to your TAS, FF point, like the one labeled "Actual SR" in Figure 82. As these tangent lines become shallower your range improves. Range improves as you climb higher and higher – in the diagram you reach optimum range at 25,000 feet but you would likely never reach that altitude in real world operations, so it is fair to say that in zero wind you should fly as high as possible.

Granting that pilots fly at power settings such as 60% to 70%, while the aeroplane requires much less for maximum range, most piston aeroplanes experience increased efficiency with altitude. **The optimum altitude for flight is the one at which maximum groundspeed for the chosen power setting is achieved.** This does not take into consideration fuel consumed in climbing to altitude. For shorter trips it will not be worth climbing to the optimum altitude, but for longer flights you should.



**Figure 82**

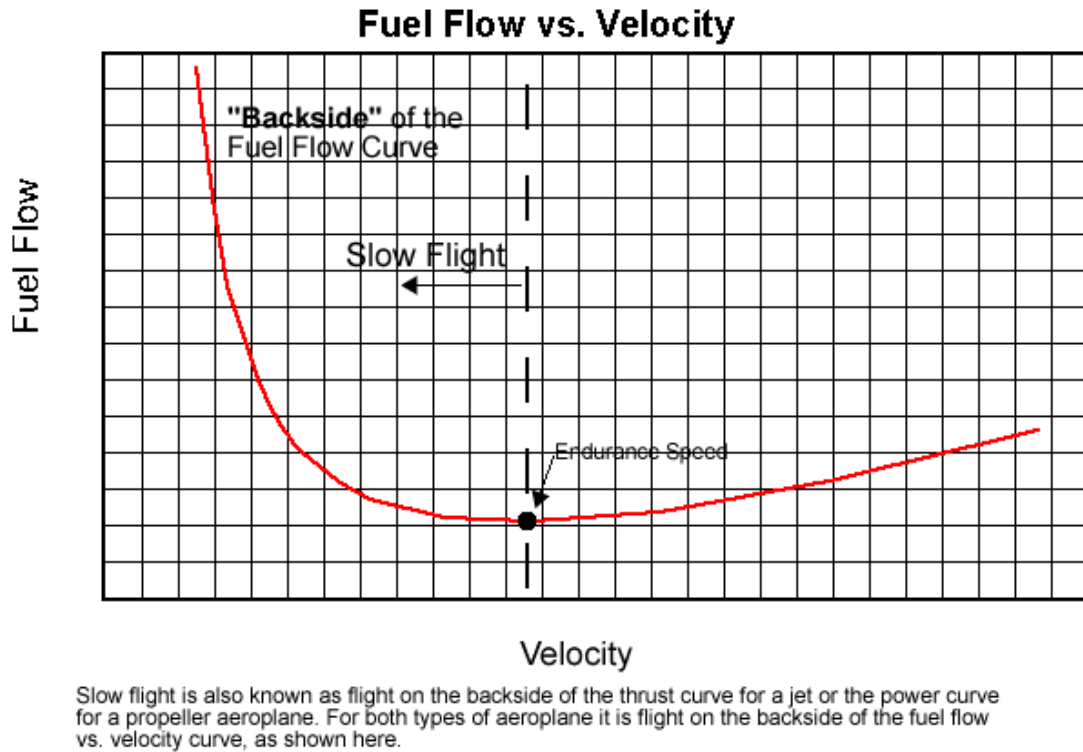
To summarize what we know about flying for maximum range in a piston aeroplane; assuming that you are committed to flying a certain power setting (such as 65%) simply determine, taking wind into account, what altitude will give the greatest groundspeed. If you have a tailwind that will certainly be a high altitude since true airspeed and ground speed both increase with altitude. But, if headwind increases with altitude you will have to calculate to see if the increased true airspeed is sufficient to compensate for the increased headwind or not. For example an aeroplane that cruises at 200 KTAS at sea level would cruise at about 270 KTAS at 20,000 feet. So if the headwind increased less than 70 knots between sea level and 20,000' you are better off to climb, but if the headwind increased by more than 70 knots it would be better stay low.

### **Slow Flight**

Slow flight is defined as flight on the *backside of the power curve* for propeller aeroplanes or flight on the backside of the drag curve for a jet aeroplane.

For both types of aeroplanes slow flight can also be defined as flight on the backside of the fuel flow curve. This is equivalent to saying that slow flight is *flight slower than the endurance speed*.

Figure 83 shows the backside of the fuel flow curve.

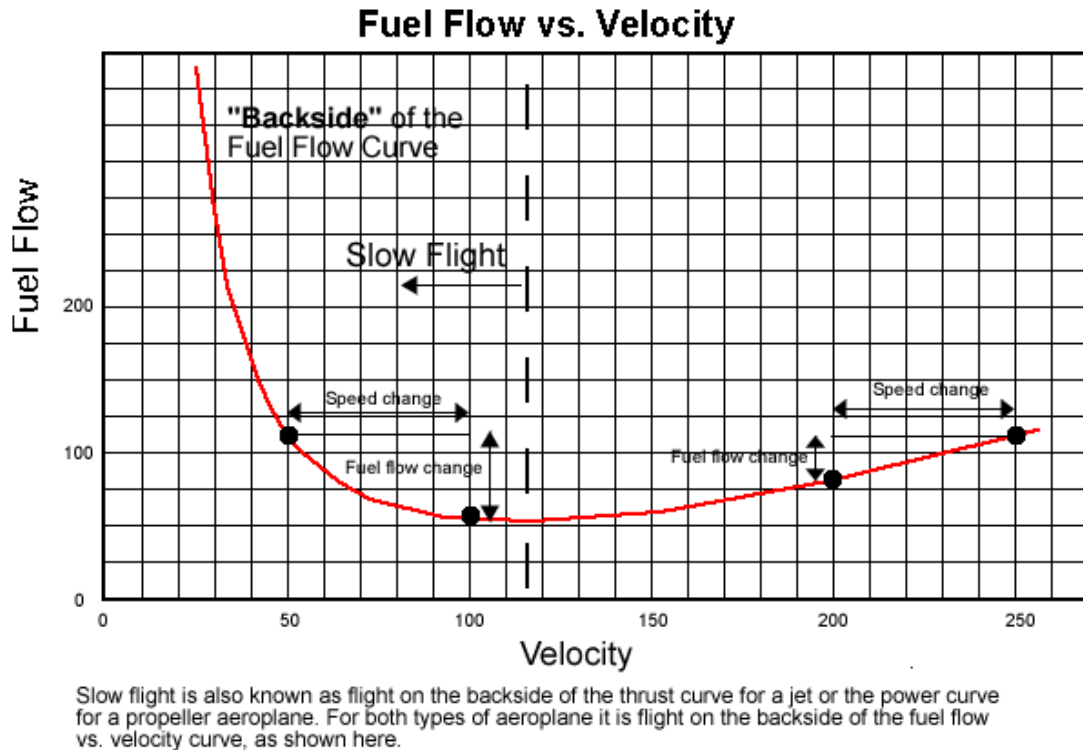


**Figure 83**

What is so special about slow flight that it deserves a special name and to be included on the private and commercial flight tests?

Consider what is involved in changing airspeed. Figure 84 shows that to slow from 250 to 200 knots the pilot of a certain aeroplane must reduce fuel flow from 115 to 80 units. That is accomplished by reducing the throttle setting. That seems sensible enough. What is required to slow from 100 to 50 knots?

The graph shows that fuel flow must actually increase from 55 to 110 units. This is “backward” from common sense; hence the term “backside of the power curve.”



**Figure 84**

To change airspeed in slow flight requires a power or thrust change that is backward from “normal.” But due to momentum you will notice that two power adjustments are actually required. For example, let’s say you want to slow from 100 to 50 knots in the aeroplane represented by Figure 84. You would initially need to reduce power slightly so that you can slow down without zooming<sup>22</sup>. Once speed has dropped you would increase fuel flow until it was more than previously set. Slow flight requires the pilot to be on his or her toes. It is excellent practice and highly recommended.

## Climb Performance

Every pilot needs to know some basic facts about aeroplane climb performance. We are interested in the amount of time it takes to climb to altitude and the horizontal distance covered during a climb. Consequently every pilot memorizes the best rate of climb speed ( $V_y$ ) and the best angle of climb speed ( $V_x$ ) for their aeroplane.

$V_y$  is the airspeed at which the aeroplane gains altitude in the least amount of time. I.E has maximum vertical speed.

$V_x$  is the airspeed at which the aeroplane gains altitude in the least horizontal distance, in zero wind.

<sup>22</sup> Remember that zooming means to trade airspeed for altitude.

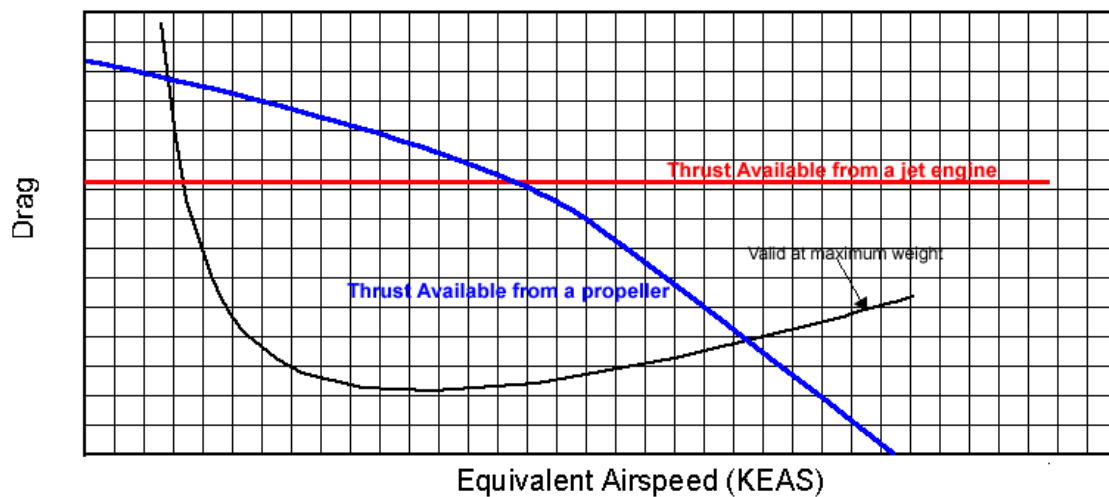
It is important to realize that wind affects the angle of climb but does NOT affect the rate of climb (i.e. does not affect time to climb.)

Refer back to Figure 25 and the explanation given at that time for the fact that angle of climb depends on thrust being greater than drag. We know that:

$$\sin(c) = (T-D) / W \quad [\text{Sine of angle of climb equals (thrust – drag) divided by weight}]$$
$$\sin(c) = T_x / W \quad [T_x \text{ is defined below as: } T_x = T - D]$$

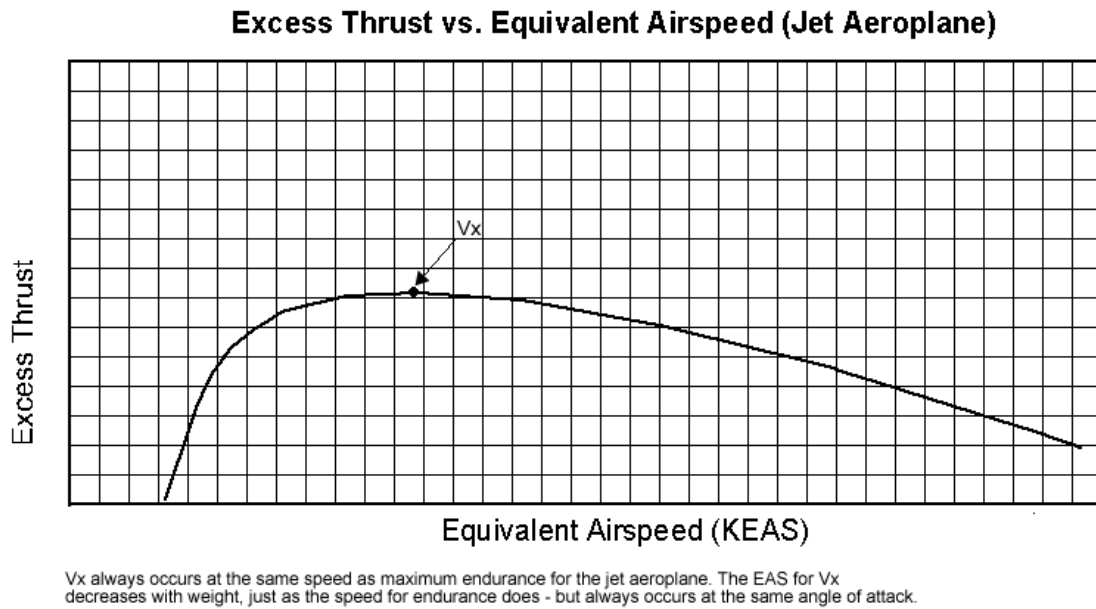
Consider Figure 85, which shows a graph of Drag vs. EAS. We already know that thrust must equal drag in level flight, so any “extra” thrust is available to make the aeroplane climb. The red horizontal line represents the thrust available from a typical jet engine at sea level. The blue line represents the thrust available from a typical propeller engine. Please realize that the values shown are examples only and would be in proportion to the size of the particular engine it is not necessarily the case that the propeller produces more than the jet, even at low speeds. The important point is that thrust is constant for a jet engine and drops off for a propeller engine.

**Drag vs. Equivalent Airspeed**



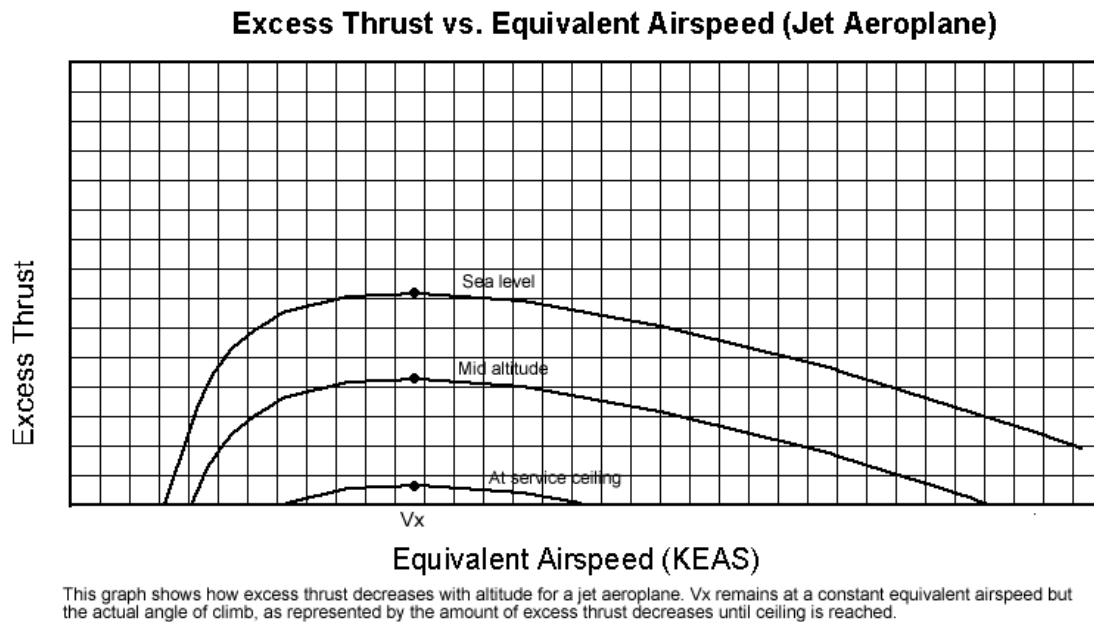
**Figure 85**

The difference between the thrust available and that needed for level flight is what we have available for climb. Figure 86 shows the “excess thrust” curve for the jet aeroplane. Excess thrust ( $T_x$ ) is defined as thrust minus drag. Notice that maximum excess thrust is always achieved at the minimum drag speed for a jet, which is the same as the maximum endurance speed. So, for a jet  $V_x$  is equal to the endurance speed.



**Figure 86**

Figure 87 shows how the excess thrust curve changes with altitude. Even though the drag curve does not change a jet engine produces less thrust at higher altitudes. At some altitude excess thrust drops off to the point that the aeroplane does not have enough thrust to climb higher. That altitude is called the **absolute ceiling**. The **service ceiling** is defined as the altitude where the aeroplane has just enough thrust to climb 100 fpm.

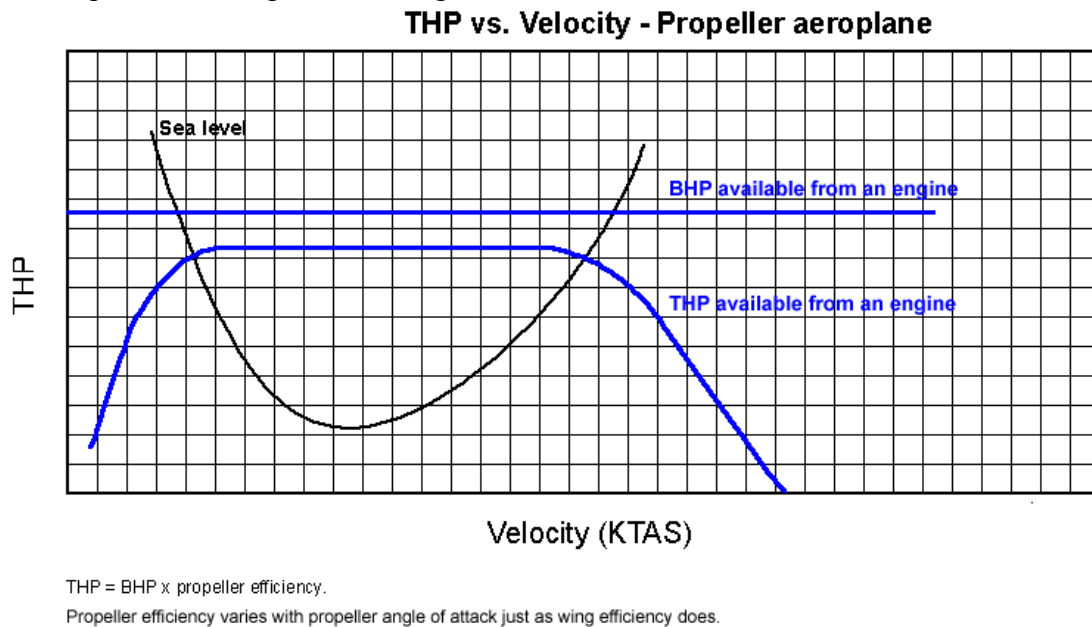


**Figure 87**



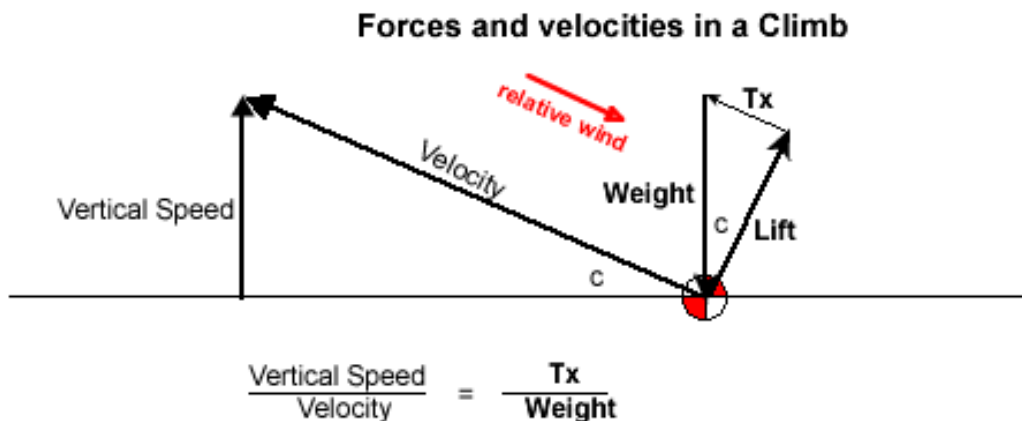
An excess thrust curve for a propeller aeroplane can also be plotted, examine Figure 85 to see what it looks like. There is no fixed relationship between the minimum drag speed and  $V_x$ . Due to the high thrust at low airspeed, for most propeller aeroplanes  $V_x$  is slower than minimum drag speed, but it could be higher, depending on the propeller design.

When analyzing climb performance for a propeller aeroplane it is usually more convenient to examine the power-required curve, rather than the drag curve.  $THP = \text{drag} \times \text{Velocity} / 325.66$  as explained on page 27. Once a THP curve is available the relationship shown in Figure 88 emerges.



**Figure 88**

We will return to Figure 88 and its significance shortly, but first consider the diagram below.



**Figure 89**

Figure 89 shows the relationship between vertical speed and climb angle. Note that rate of climb is commonly called vertical speed by pilots. It is crucial for pilots to realize that vertical speed is simply the *vertical component of true airspeed i.e. velocity*, as shown in the figure. Consequently two aeroplanes climbing at the same angle of climb do not have the same vertical speed unless they also have the same velocity. The faster aeroplane has a greater vertical speed. From the figure we can see the triangle created by the lift, weight and  $T_x$  vectors is similar to the triangle containing vertical speed and velocity. Therefore by the rules for similar triangles it follows that:

$$\text{Vertical speed} / V = T_x / W$$

Therefore:

$$\text{Vertical speed} = T_x \times V / W \times 6080/60 \quad [6080/60 \text{ converts vertical speed to units of ft/min}]$$

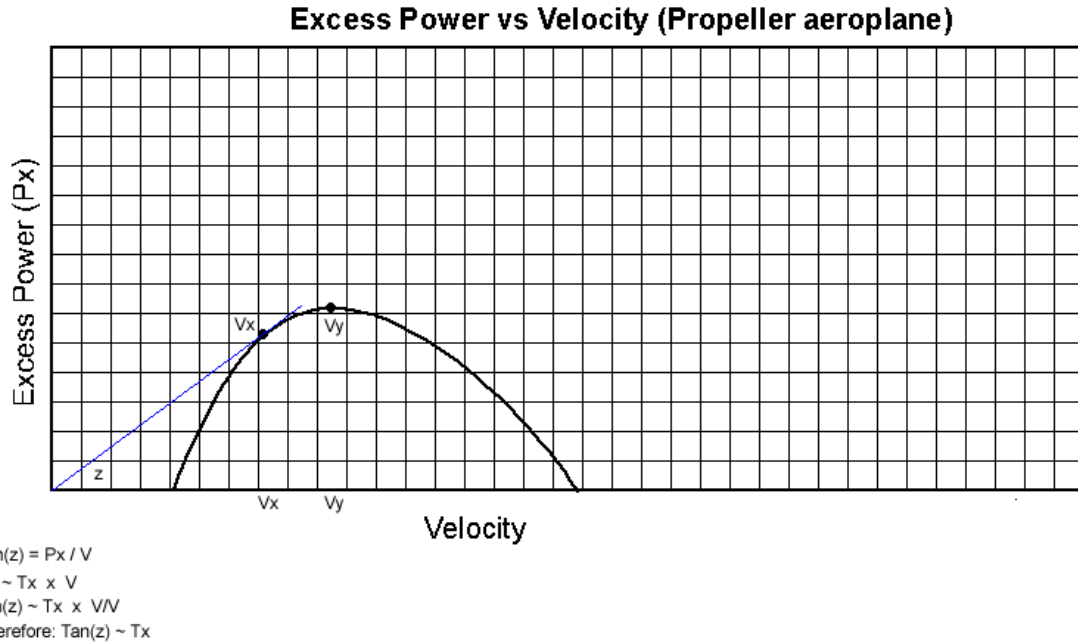
Recall that we previously learned that power is:  $\text{THP} = T \times V / 325.66$ . We will now introduce a new term called excess power ( $P_x$ ) which we will define as:

$$P_x = T_x \times V / 325.66$$

$P_x$  represents the excess power, i.e. the amount of power available from the engines beyond that needed for level flight. Now we can summarize the vertical speed equation in two equivalent forms:

1. Vertical speed =  $T_x \times V / W \times 6080/60$
2. Vertical speed =  $P_x / W \times 6080/60 \times 325.66$

We can now return to Figure 88 and obtain an excess power graph by subtracting power required from THP available. It is important to realize that we must use THP not BHP because only the power actually delivered by the propeller is available to make the aeroplane climb.



**Figure 90**

The excess power curve for a propeller aeroplane looks like the one shown in Figure 90. The point at the top of the curve is obviously  $V_y$ . It is less obvious that the point marked by the tangent line, as shown, is  $V_x$  (It is the point at which the ratio  $P_x/V$  is maximum. But that amounts to saying that  $T_x \times V / V$  is maximum i.e. that  $T_x$  is maximum. As we know  $V_x$  occurs when  $T_x$  is maximized.)

Notice that in both jet and propeller aeroplanes  $V_y$  is always greater than  $V_x$  (see next paragraph for changes with altitude.)

Figure 91 shows how the excess-power curve changes with altitude. For propeller aeroplanes  $V_y$ , in equivalent airspeed, usually decreases slightly with altitude while  $V_x$  always increases with altitude. For jet aeroplanes it should be clear that  $V_x$  in equivalent airspeed remains constant and therefore  $V_y$  must decrease with altitude. In both types of aeroplane  $V_x = V_y$  at the absolute ceiling.

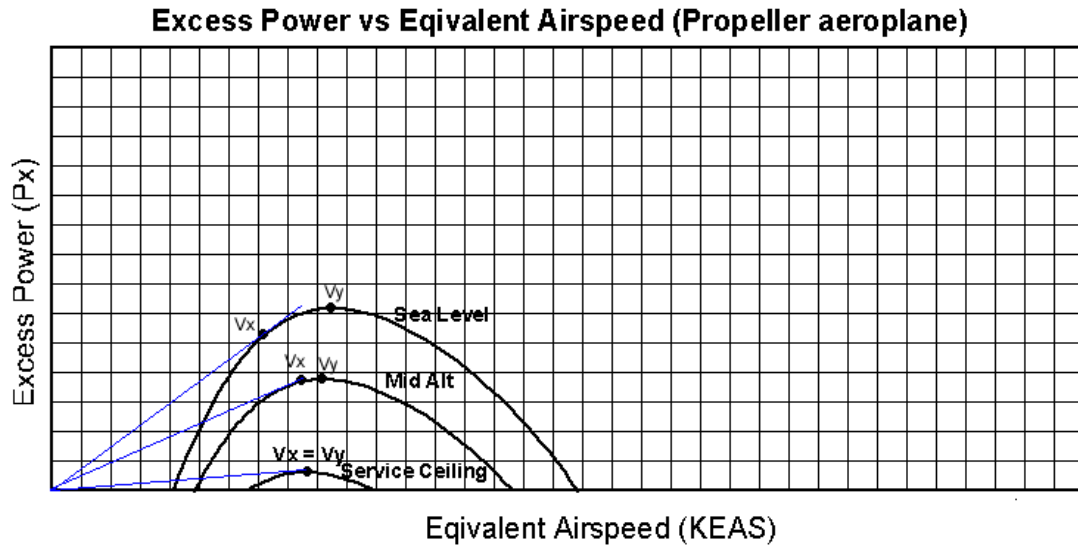


Figure 91

## Glide Performance and the Lift/Drag Ratio

### Gliding Performance and the L/D Ratio

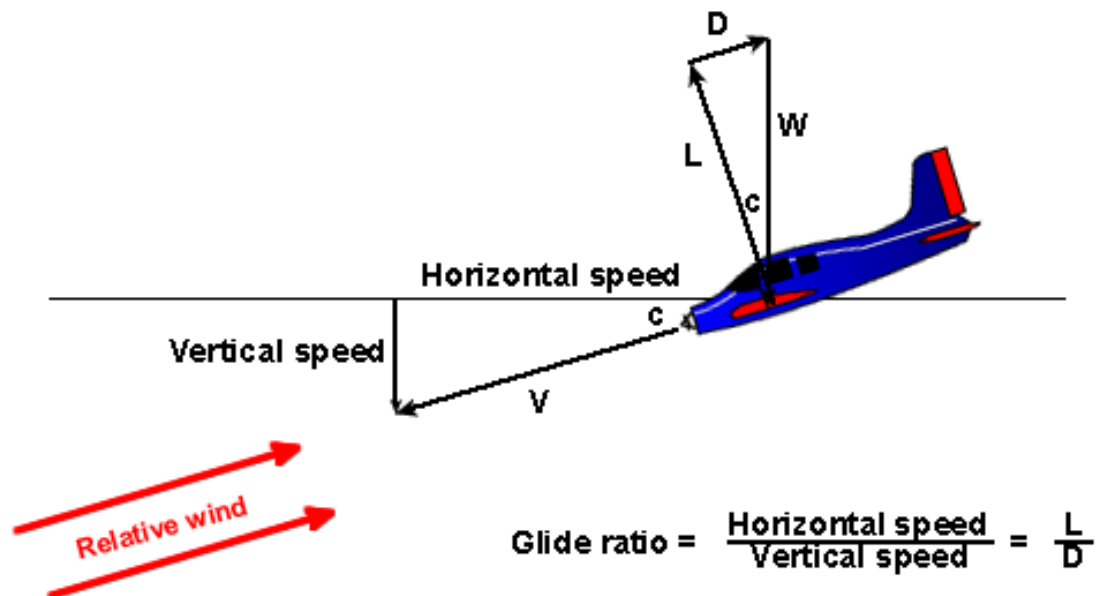


Figure 92

Figure 92 shows the three forces acting on an aeroplane that is gliding. The only forces are Lift, Weight, and Drag, because thrust equals zero. The figure also shows the velocity

vector. Remember that lift is perpendicular to this vector while drag is parallel but opposite to it.

Vertical speed and horizontal speed are the two components of velocity, as shown. The ratio of horizontal speed to vertical speed is called the **glide ratio**. It expresses how many feet forward the aeroplane travels for each foot of altitude lost during a glide.

The triangle formed by horizontal speed, vertical speed, and velocity is similar to the triangle formed by Lift, Drag, and Weight. Thus the ratio horizontal speed / vertical speed exactly equals the ratio Lift / Drag.

To obtain maximum glide in zero wind an aeroplane must be flown at maximum L/D ratio, which is the same as saying minimum drag. On the previously examined drag curve (see Figure 63) that would be the bottom of the curve.

Note that if you glide at a speed other than minimum drag the glide ratio still equals the L/D ratio, but the ratio is no longer the maximum possible ratio. This is true whether you glide faster or slower than minimum drag speed.

### ***The L/D Ratio***

The L/D ratio is calculated by dividing lift by drag. In other words:

$$\frac{L}{D} = \frac{C_L \times S \times 1.426 \rho V^2}{C_D \times S \times 1.426 \rho V^2}$$

$$\frac{L}{D} = \frac{C_L}{C_D}$$

**But:**

$$C_D = C_{Dp} + C_{Di} =$$

**Therefore:**

$$\frac{L}{D} = \frac{C_L}{C_{Dp} + \frac{C_L^2}{\pi e AR}}$$

From the equation we can see that the only factors contributing to L/D ratio are  $C_L$  (i.e. angle of attack) and  $C_{Dp}$  (which changes when flaps and gear settings are changed.) The other factors ( $e$  and  $AR$ ) are design parameters and thus beyond pilot control.

Maximum L/D occurs at minimum  $C_{Dp}$ , i.e. with flaps and gear up, and at a certain *specific angle of attack*. This angle of attack should be used regardless of weight. It is a fact worth noting that in zero wind *a heavy aeroplane can glide just as far as a light aeroplane*, but both must fly at the same angle of attack; the heavy aeroplane will therefore glide at a faster airspeed.

### ***Gliding into a Wind***

Wind affects glide range just as it affects cruise range. An aeroplane cannot glide as far into a headwind, but glides farther with a tailwind.

All things being equal, glide is best analyzed with a Power vs. Velocity graph such as Figure 78. Maximum glide speed in zero wind is at the point where the tangent line from the origin touches the curve, just as in Figure 81, and if there is wind the glide-speed should be adjusted as in Figure 76. The conclusion seems to be that zero wind best glide speed for a propeller aeroplane is the same as the best range speed in level flight and for a jet aeroplane is the same as the best endurance speed. This is true if “things are equal”, but usually things are not quite equal. The L/D formula shows that glide ratio depends on  $C_{Dp}$  and this value often changes following an engine failure, increasing substantially due to the propeller wind-milling. When  $C_{Dp}$  increases the L/D ratio decreases and the optimum angle of attack increases (i.e. the aeroplane must glide at a lower speed.) Consequently the best glide speed is usually somewhat slower than the best range speed with engines operating, and this effect is greatest for aeroplanes with fixed pitch propellers.

It is important to note that when gliding into a headwind *glide speed should be increased*. The graphical proof is exactly like that for cruising with a headwind shown previously. With a headwind the aeroplane should glide at a lower angle of attack. This leads to the counter intuitive situation in which a gliding pilot turning toward a desired touchdown point, and presumably into a headwind, who finds the aeroplane gliding short must *lower the nose* to glide farther. All pilots should experiment with this phenomenon as part of flight training as it could save you some day.

With a tailwind maximum glide range is achieved at a slightly lower speed, i.e. higher angle of attack than in zero wind. Knowing this is theoretically important but seldom worth using in reality.

Since glide ratio depends on L/D ratio, anytime a pilot wants to descend more steeply while gliding the objective is to increase drag as much as possible. Adding flaps will do the trick, as will extending gear, spoilers, or any other drag producing devices. Increasing speed (i.e. diving) will increase drag and result in a steeper descent, but less obvious is that raising the nose will also increase drag (induced drag) and cause the aeroplane to

sink faster. A sideslip is also effective as it increases drag by destroying the streamline nature of the aeroplane.

### ***Gliding for Endurance***

We usually think of gliding as an emergency procedure used by pilots of powered aeroplanes following an engine failure. In such cases gliding for maximum range is usually the objective.

Gliding for maximum endurance means maximizing time in the air, i.e. descending with the minimum possible vertical speed. For glider pilots this is an important concept as it optimizes the amount of altitude they can gain in an up-draft. Maximum endurance glide speed is always slower (higher angle of attack) than for best range. On a Power vs. Velocity graph it is at the bottom of the curve and should in theory correspond to the same speed as maximum endurance in level flight. As explained under gliding for range above that is true only if the coefficient of parasite drag is the same during glide as it is in cruise. It will always be the case that maximum endurance glide occurs at a slower speed than maximum range. Knowing this could be useful even to a powered aeroplane pilot who is seeking time to perform a drill such as an engine re-start procedure.

## Stability and Control

How stable would you like your aeroplane to be?

Many pilots would say, “as stable as possible” but that would be a mistake. Stable is the opposite of controllable. If an aeroplane is too stable it is not controllable. But if it is not stable enough it will go “out of control.” So the topics of stability and control are always studied together.

The first aeroplane to fly, the Wright flyer, was unstable. The first attempted flight, on December 14, 1903 ended in a stall and crash at liftoff. Hence the first **successful** flight was on December 17 (after repairs.) By the way, the instability of the flyer was an intentional design feature. The Wright brothers considered control to be more important than stability, so they deliberately designed the aeroplane that way. Today the CARs would not permit that.

What exactly do we mean by stability?

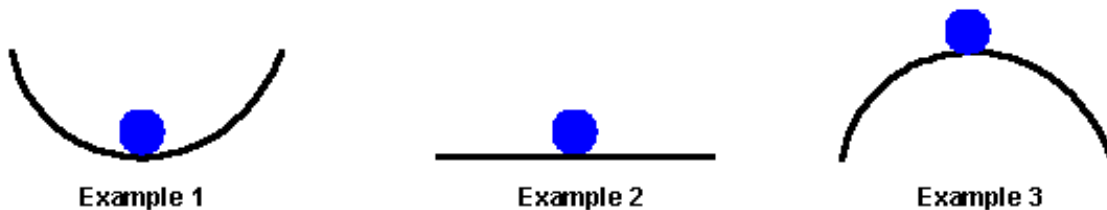


Figure 93

Figure 93, example 1, shows a marble in a bowl. The marble is stable, meaning that if you push it, it will roll back to the original spot. The marble in example 2, on the flat floor, has neutral stability, meaning that if you move it, it will remain wherever you put it, neither returning nor moving farther away. In example 3 the marble, balanced on top of the inverted bowl, is unstable. If it moves, even a little, it will roll farther away.

In the case of the marbles above it is clear that we are talking about the *location* of the marble when we say they are stable or not. But when we say that an aeroplane is stable what aspect of the aeroplane are we talking about? We are talking about its orientation to the relative wind, i.e. angle of attack, angle of slip, and bank attitude.

When examining aeroplanes we must consider stability about all three axes. We will discuss:

1. Directional Stability; response to slip caused by rotations around the normal axis.
2. Longitudinal Stability; response to angle of attack changes caused by rotations around the lateral axis.



3. Lateral stability: responses to bank caused by rotations around the longitudinal axis.

## Static and Dynamic Stability

Static stability refers to how the aeroplane initially responds to a change. Static stability can be positive, neutral, or negative, as exemplified by the marbles above. The marble examples dealt with the static stability of the marble.

Dynamic stability examines the oscillations that occur over time. An aeroplane that has positive static stability will often oscillate back and forth following a disturbance. Take the marble example again. The marble in the bowl, if displaced, will roll back to the bottom, but it will overshoot and “slosh” back and forth several times before friction stops it and it comes to rest at the bottom of the bowl. Provided that friction stops it we say that the marble has positive dynamic stability. **Dynamic stability is only analyzed if an aeroplane has positive static stability.**

Is it possible that the marble in the bowl could have negative dynamic stability?

It seems that friction is bound to stop the marble so there should to be no way it could have anything other than positive dynamic stability. But if the marble is displaced during an earthquake that shakes the bowl the marble could be bounced up higher, or even out of the bowl. The point is that if energy is added then negative dynamic stability is possible. In the case of an aeroplane negative stability would likely be due either to energy put in by the engines, or miss-controlling by the pilot, resulting in growing oscillations and eventual loss of control. Negative dynamic stability is unusual, but possible. Full analysis of dynamic stability is beyond the scope of this text. The discussion that follows concentrates on static stability. It turns out that pilots can successfully fly an aeroplane with some dynamic instability as long as static stability is within normal parameters.

## Directional Stability

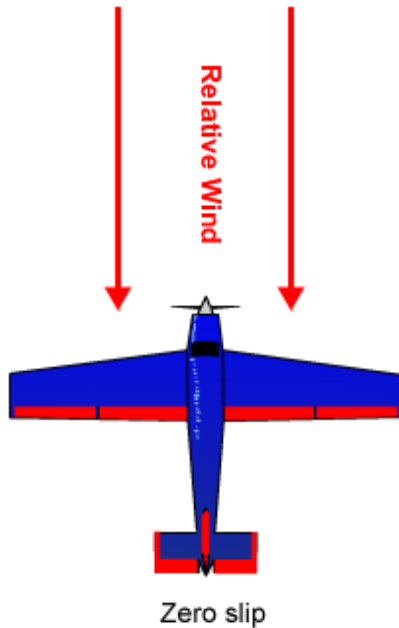


Figure 94

Directional stability is the easiest to understand. Figure 94 shows a top view of an aeroplane with the relative wind flowing parallel to the longitudinal axis. By definition the slip angle is zero. What happens if the pilot gives the rudder a “kick” then lets go?

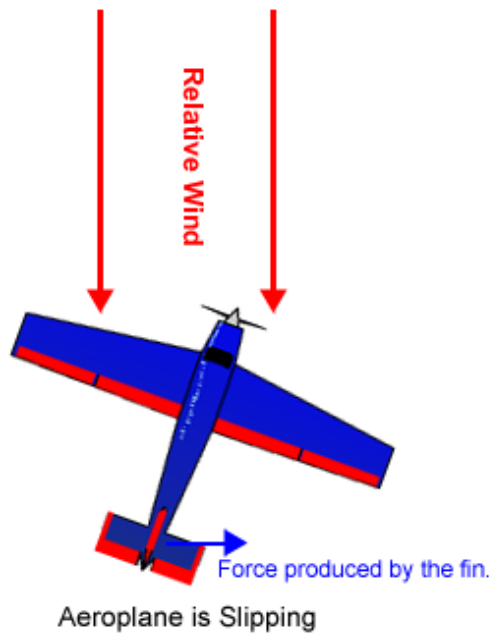


Figure 95

Figure 95 shows the aeroplane just after the pilot releases the right rudder. The rudder has returned to neutral and the relative wind strikes the fin at an angle of attack. Like any

wing the fin produces lift and the lift creates a moment that tends to bring the aeroplane back in line with the relative wind.

Because the aeroplane has momentum it will likely overshoot and oscillate back and forth for a few seconds, but most aeroplanes will very quickly settle back to flight at zero slip again.

In a nutshell the above explains why aeroplanes have directional stability. Keep it in mind, we will return to it. It is commonly called “weather vane” tendency, because it is the way weather vanes and windsocks work. In summary, **the fin gives an aeroplane static directional stability**.

What would happen if the pilot applied and held a small amount of rudder? The answer is that the aeroplane would establish a slip angle. For every rudder deflection there is a unique slip angle, i.e. more rudder means larger slip angle. The slip angle is unrelated to airspeed, it depends only on how much rudder is applied. Before you try experimenting with this however read about dihedral effect below.

## Longitudinal Stability

Applying rudder to maintain a slip angle is a fairly unusual thing for a pilot to do, except perhaps when making a crosswind landing. But pilots do use the elevators to control angle of attack. For some reason pilots find it quite easy to visualize the weather vane tendency as it applies to directional stability but often don't recognize that the exact same considerations apply to longitudinal stability.

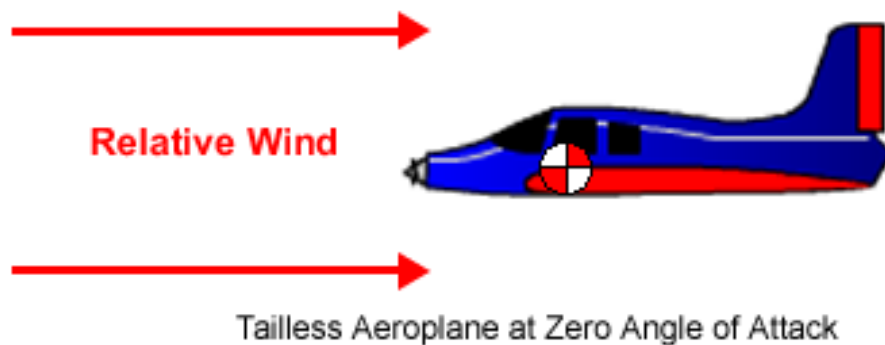


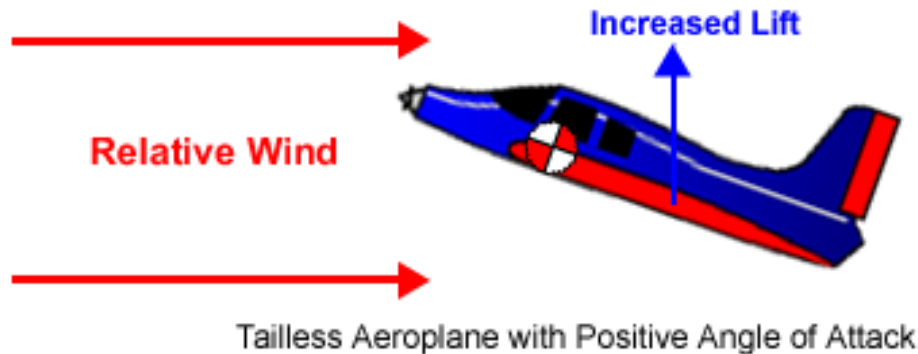
Figure 96

Perhaps the thing that complicates longitudinal stability in pilots' minds is that there are two wings rather than just one in this axis. So, we will start with an aeroplane that has no horizontal tail. Such an aeroplane is said to be tailless. Figure 96 shows a tailless aeroplane flying at zero degrees angle of attack<sup>23</sup>. An example of this would be the

<sup>23</sup> As drawn this aeroplane could not fly because the center of pressure would not line up with the CG. A tailless aeroplane would require a reflexed airfoil such that CP and CG coincide.

Concorde jet. What happens if the pilot deflects the elevators momentarily then releases them?

Figure 97 shows the situation just after the angle of attack was increased. As long as the increased lift occurs behind the CG the situation is *identical* to the directional stability case covered earlier; the aeroplane is forced back to the original angle of attack (zero in the example.) We can see then that as long as the CG is well ahead of the lift vector the airplane will have positive static longitudinal stability.



**Figure 97**

To get the aeroplane to fly at a desired angle of attack the pilot must deflect the elevator controls; the trim tab can then be used to persuade the aeroplane to remain at that new angle of attack (we cover trim tabs shortly.) The important point to remember is that once trimmed *a stable aeroplane will return to its trimmed angle of attack if disturbed.*

The fact that most aeroplanes have both a main wing and a horizontal tail doesn't actually change anything said above. Don't let the two-wing complication throw you off, what you need to know is that once you set the elevator trim the aeroplane always returns to the trimmed angle of attack if disturbed, although it may overshoot and oscillate a few times before settling down.

Since stability depends on the CG being ahead of the Lift vector it follows that the stabilizer at the rear of the aeroplane is very effective at providing longitudinal stability. Indeed that is why it is called a stabilizer. The larger the stabilizer, or the farther aft it is the stable the airplane will be. It should also be obvious that loading the aeroplane with a forward CG will increase both directional and longitudinal stability.

*A small percentage of aeroplanes may experience increasing oscillations and not settle down; In other words they are dynamically unstable. Normally such aeroplanes are not certified; even so it is usually possible to fly a dynamically unstable aeroplane as long as it is statically stable. Static stability provides the important cues pilots need to keep from over controlling.*

The CARs specify the minimum stability requirements for all aeroplanes. Pilots can rest assured that as long as the CG is kept within the specified limits the aeroplane will have sufficient static stability to be flyable.

### Trim tabs

Figure 98 shows how a trim tab works. Both elevator and rudder trim tabs work the same way. The tab is in effect a small wing that uses its lever arm to hold the control (elevator or rudder) in the deflected position. The tab simply provides the force to deflect the control surface that otherwise would have to be provided by the pilots muscles.

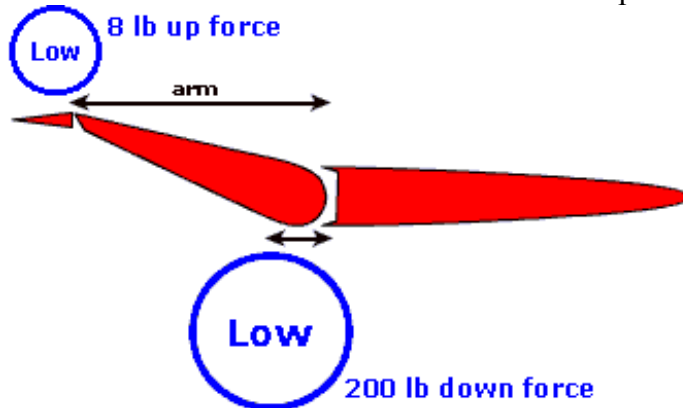


Figure 98

You can rest assured that once you trim an aeroplane it will continue to fly at the same slip angle (rudder trim) or angle of attack (elevator trim) *unless you change the power setting or configuration* of the aeroplane. We will discuss that shortly.

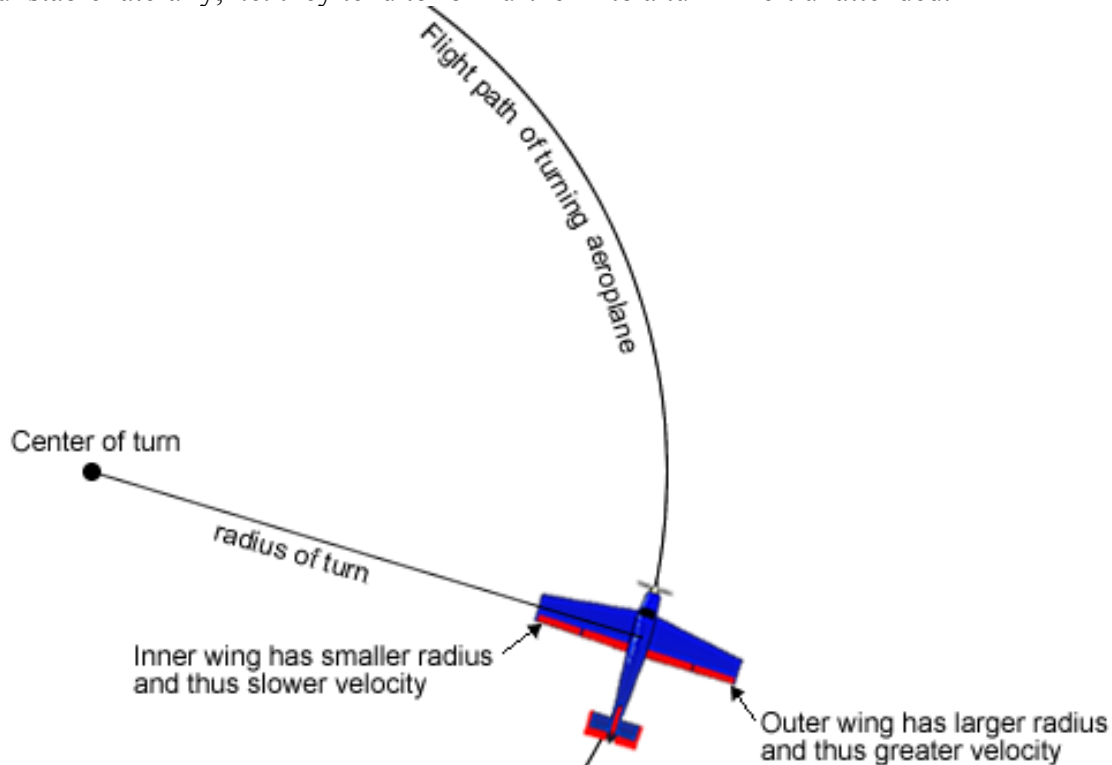
### Lateral Stability

Lateral stability refers to how the aeroplane responds to rotation around the longitudinal axis. This is quite different than longitudinal and directional stability. Can you see why?

The longitudinal axis is the only one that has airflow parallel to it. Consequently there is no reason why the aeroplane should prefer to fly wings level as opposed to any other bank attitude. Take for example a dart that you might throw at a dartboard. It probably has three feathers, but does it matter which one is up, or if any of them is exactly up? No. So why would your aeroplane care whether the fin is up, down, or sideways? The answer appears to be that it wouldn't.

You have probably noticed that you can place an aeroplane in a banked attitude and release the controls - for the most part the aeroplane turns and remains banked. It neither rolls further in, nor does it roll out. The aeroplane has approximately *neutral static lateral stability* (like a dart.)

There are a few differences between an aeroplane and a dart however. The most obvious is that an aeroplane tends to turn if in a banked attitude, while a dart doesn't. Figure 99 points out that when an aeroplane turns the outer wing flies faster than the inner wing. Consequently it will produce slightly more lift. This tends to make all aeroplanes slightly unstable laterally, i.e. they tend to roll further into a turn if left unattended.



**Figure 99**

If you throw a football you normally try to establish a rotation as you release it. But if you try to start an aeroplane rotating you notice that it stops as soon as you return the ailerons to neutral. This at first seems to violate rule 1, so why does this happen? It is the result of **roll damping**.

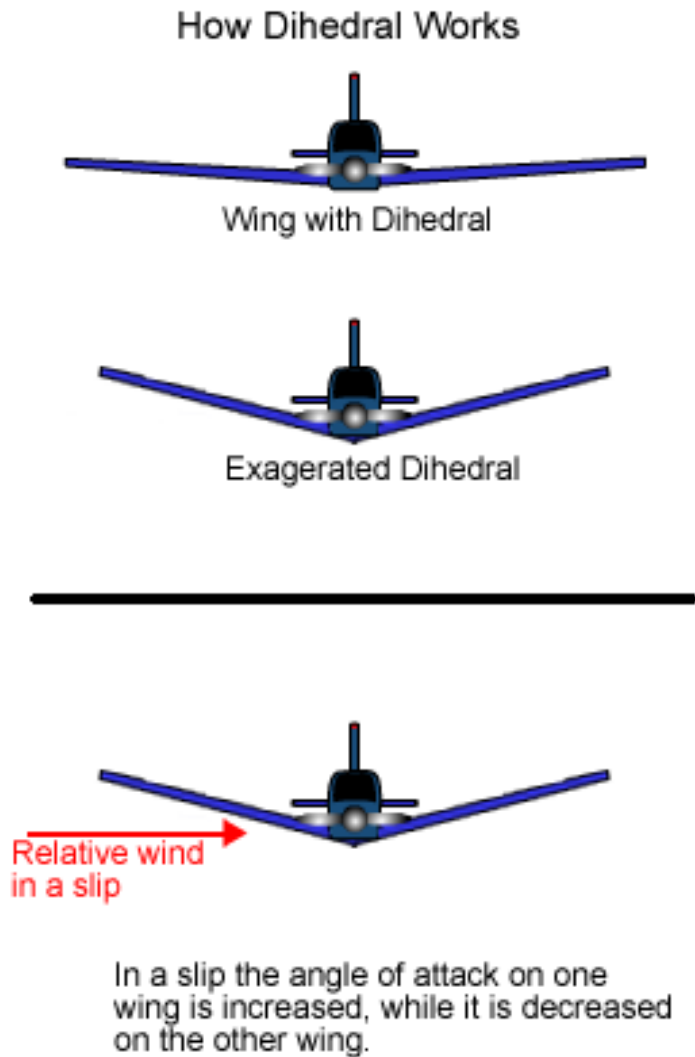
When an aeroplane is rolling the down going wing strikes the air at a greater angle of attack than the up going wing. This increases lift on the down going wing and reduces it on the up going wing. Consequently ailerons must be kept deflected in order to maintain a constant roll rate. A small aileron deflection produces a small roll rate, and a large aileron deflection produces a large roll rate. As soon as the ailerons are neutralized the roll rate rapidly drops to zero. The aeroplane then maintains approximately the established angle of bank, although it may tend to roll slowly into the turn due to the greater velocity of the outer wing, as already explained. Roll damping is effective at stopping increases in angle of bank but is of no help in returning the aeroplane to wings level.

We started this section off by pointing out that air normally flows parallel to the longitudinal axis and that is why lateral stability is hard to establish, but if the aeroplane

slips the situation changes completely. All aeroplanes have a roll response to slip, i.e. they roll when slipped. There are three design parameters that determine the extent of the roll response to slip:

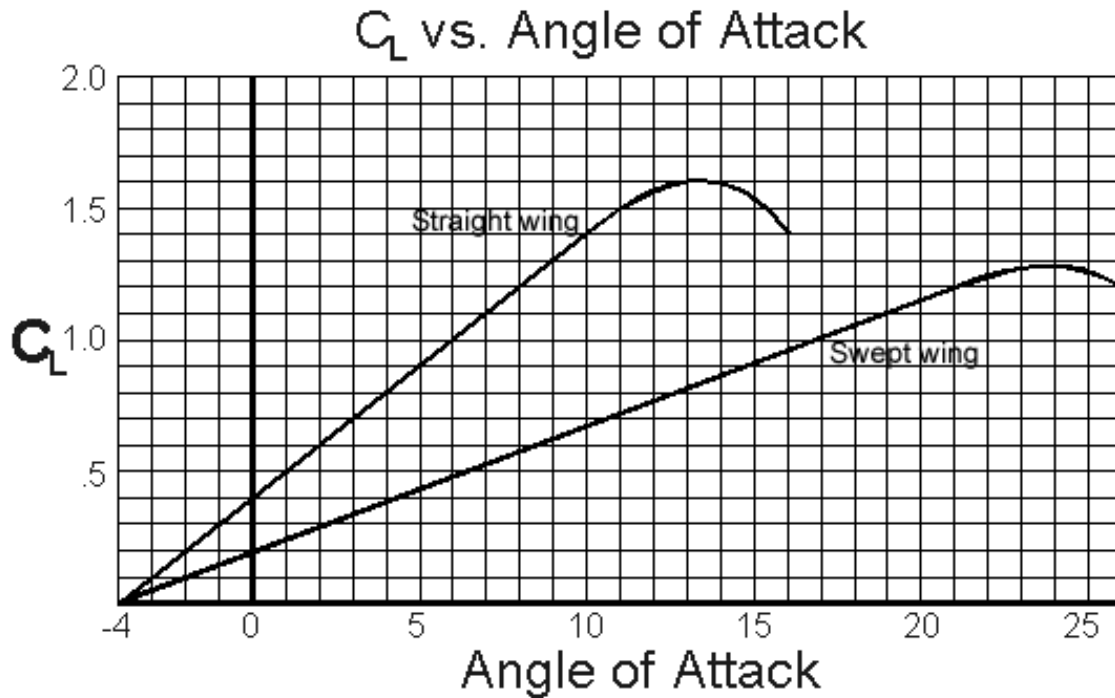
1. Dihedral
2. High vs. low wing
3. Wing sweep

Figure 100 shows wing dihedral. If an aeroplane has dihedral it will tend to roll opposite to a slip. I.E. it will roll in the direction that rudder is applied. The reason is that once the aeroplane slips the air flows across the wing at an angle. This increases the angle of attack on one wing and reduces it on the other wing. The lift difference causes a roll.



**Figure 100**

High wing aeroplanes respond the same as aeroplanes with dihedral. The drag caused by the slip, which acts opposite to the direction of flight, creates a rolling moment in the direction that rudder is applied. Note that the opposite is true for low wing aeroplanes, so you will notice that low wing aeroplanes always have a lot more dihedral than high wing aeroplanes. In addition, air deflected upward by the fuselage side of a high-wing aeroplane increases angle of attack on the side of the slip.



**Figure 101**

Swept wings also affect roll response to slips. Figure 101 reminds us how  $C_L$  vs.  $\alpha$  varies for straight and swept wing aeroplanes. Figure 102 shows that in a slip one wing is in effect straighter, while the other is more swept. Consequently the lower wing produces more lift, rolling the aeroplane in the direction opposite the slip.



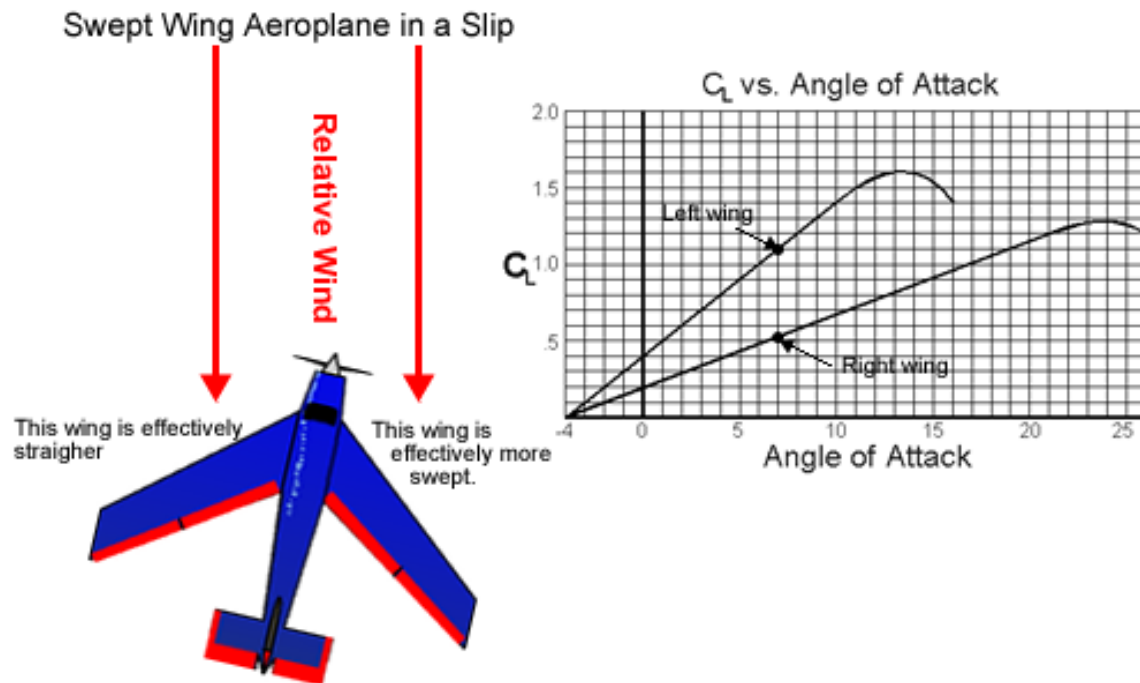


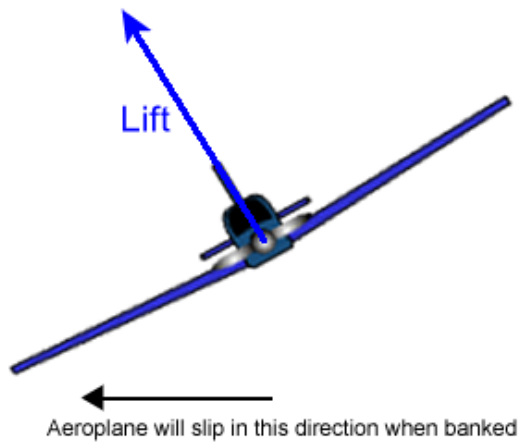
Figure 102

All three effects discussed above contribute to what engineers call the **dihedral effect**. Note that it is called *dihedral effect* even if its cause is actually high or swept wings. To test how much dihedral effect your aeroplane has, trim it for level flight then let go of the control column. Control angle of bank with gentle rudder pressures. In almost all light aircraft you can easily roll the aeroplane back and forth with the rudder. You can stop bank at any desired amount, then reverse the roll etc. You can also keep the wings level with the rudders. If your aeroplane does this it has dihedral effect, in other word positive static later stability. Some aeroplanes have more dihedral effect than others. Swept wing aeroplanes usually have, by far, the greatest dihedral effect. It is often so strong that jet pilots are advised to “keep feet off the rudders.”

## Why an Aeroplane Turns

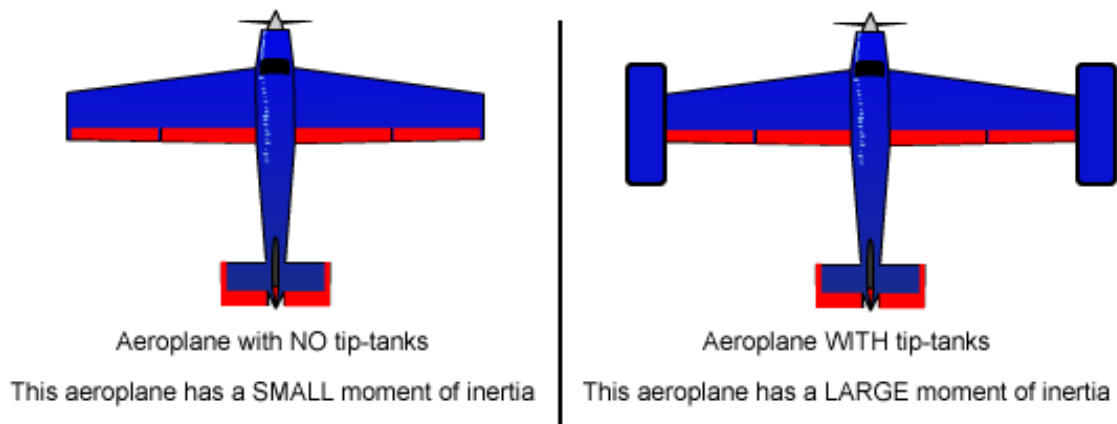
Every pilot knows that if you place an aeroplane in a banked attitude it turns; but why?

Figure 103 shows that in a bank the lift vector is automatically inclined with the bank. This is because the air molecules supporting the wing can only push directly on the wing surface (rule 4.) The inclined lift vector tends to move the aeroplane sideways. In other words it tends to make the aeroplane slip! The slip quickly starts the aeroplane turning, as explained in the discussion about directional stability above.



**Figure 103**

Because a bank always initiates a slip the dihedral effect has an opportunity to roll the wings level again. How effective it is depends on how strong the dihedral effect, and how weak the directional stability is. The other factor is the momentum of the aeroplane, or more precisely the moment of inertia about the normal axis. An aeroplane with a lot of weight distributed away from the CG, as in Figure 104, will take longer to start turning and thus will slip for longer. The dihedral effect will work more on such an aeroplane than one that has its weight concentrated near the center and has limited dihedral effect to begin with.



**Figure 104**

The consequence of the above is that aeroplanes tend to fall into two broad categories: some are spirally divergent, while others suffer from Dutch roll tendency. We will now examine each of these phenomena.

## **Spiral Divergence (Spiral Dive)**

If an aeroplane has lots of directional stability and limited dihedral effect it will be spirally divergent. This description applies to almost all light aeroplanes.

If a spirally divergent aeroplane is trimmed to fly straight and level and then left unattended inevitably the following series of events will take place:

1. Random turbulence causes a slight bank angle.
2. The bank starts a slight turn.
3. The turn necessitates more lift.
4. Because the aeroplane is longitudinally stable the angle of attack cannot change, so the aeroplane “drops” due to insufficient lift.
5. The drop increases the angle of attack but longitudinal stability causes the nose to drop maintaining angle of attack.
6. The outer wing travels faster than the inner wing, so bank angle slowly increases.
7. Steps 3 to 6 repeat over and over

The end result of the above series of actions is that the aeroplane finally winds up in an extremely steep dive at very high airspeed, high bank, and high load factor. It either breaks up due to over stressing or hits the ground at high speed.

The above maneuver is called a spiral dive. To recover the pilot should remove power, roll the wings level, then slowly raise the nose without overstressing the aeroplane. A spirally divergent aeroplane cannot recover on its own. If the pilot does not act the aeroplane will be lost.

## **Dutch Roll**

While light aeroplanes usually tend to spiral, most swept wing transport aeroplanes have the opposite problem. They have a lot of dihedral effect, due to their swept wings, and as a result they don't tend to spiral they suffer instead from Dutch roll. The sequence of actions leading to a Dutch roll is:

1. Random turbulence causes a slight bank.
2. The bank starts the aeroplane slipping toward the bank
3. The dihedral effect quickly starts airplane rolling in the opposite direction to the slip.
4. A turn (yaw) finally starts but is lagging behind the roll, so by now the aeroplane is rolling the other way.
5. Inertia causes bank to overshoot, and thus a slip in the opposite direction begins.
6. Steps 2 to 5 repeat, sometimes damping out, but often each oscillation is larger than the one before

In a Dutch roll *yaw lags behind roll*.

Dutch rolls will make the passengers sick pretty fast, so every effort is made to prevent them. Since they are caused at their root by a slip many jets have a slip sensor connected

to a device that prevents slip by quickly applying rudder. This device is known as a **Yaw dampener**. With the yaw dampener on Dutch Roll is unlikely to occur.

Dutch roll is more prevalent at high altitude where thinner air is less effective at damping it. Dutch roll is also more likely when the aeroplane slows down because directional stability will be reduced. Consequently if a Dutch roll is encountered pilots should turn on the yaw dampener if not already on, speed up if permitted, and descend to a lower altitude.

Specific procedures for recovering from a Dutch roll are provided in most transport aeroplane operating manuals, it is important to follow the prescribed procedure. The generic procedure is to leave the rudder alone and make one large stab with the ailerons opposite to the roll on each cycle until control is regained.

## **Snaking**

Snaking is similar to Dutch roll but in snaking it is roll that lags behind yaw. Snaking happens with airplanes that have a lot of moment of inertia around the normal axis. High moment of inertia exists when a lot of mass is distributed outboard from the CG. For example airplanes with tip tanks (full tip tanks) have a lot of moment of inertia. Such airplanes tend to keep yawing once they start. The yaw causes a skid which causes a roll. Eventually the skid also causes a yaw in the opposite direction and thus the oscillation repeats. If the airplane has a lot of lateral stability the snaking may be indistinguishable from a Dutch roll, and the remedy is the same. But for an airplane with less lateral stability, such as a straight-wing airplane, the snaking often takes the form of an annoying tendency for the heading to swing back and forth a few degrees. If the snaking is only two or three degrees, as is often the case the airplane is perfectly flyable and pilots will find there is little they can do about it from a practical point of view. If the snaking is more pronounced the airplane will require a yaw dampener

It is worth knowing that the directional stability of an airplane is generally greater if the rudders are held centered than if they are allowed to float. Thus the best cure for an airplane that is snaking is often to put your feet on the rudder pedals and hold them steady (i.e. don't let them float back and forth as they will do if you put your feet on the floor.)

Snaking is more pronounced at low true airspeeds and high altitudes (just like Dutch rolls.)

Some airplanes snake more at high angle of attack, usually because a portion of the fin experiences turbulent flow (from the wing or fuselage) and becomes less effective.

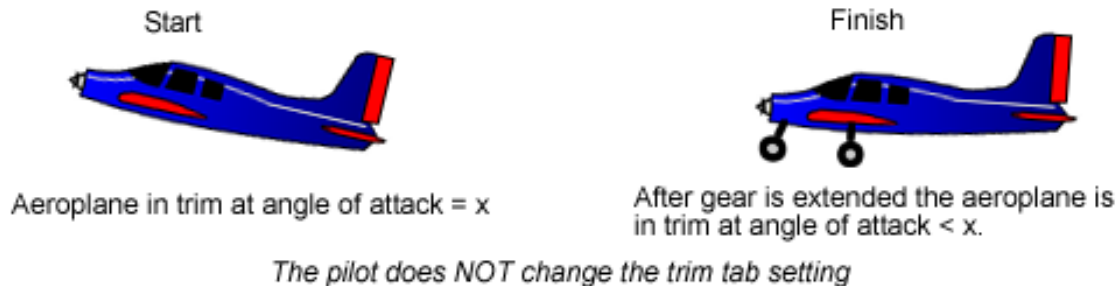
## ***Effect of Thrust on Trim Angle of Attack***

Earlier we learned that elevator trim establishes a certain angle of attack. If the pilot refrains from moving the elevator control the aeroplane will remain at the trimmed angle

of attack. There are two critical exceptions to this that pilots must know about and allow for in order to fly precisely:

1. Changing configuration (flaps or gear) changes trim angle of attack
2. Changing power/thrust changes trim angle of attack

Consider Figure 105 in which the gear has just been extended on a previously in trim aeroplane. Since the gear is below the CG its drag will create a moment that pitches the nose down and reduces the *trim angle of attack*.



**Figure 105**

Figure 106 shows that as the angle of attack decreases the down force on the tail increases and soon a new equilibrium is reached. In other words extending the gear is like trimming nose down. You can try an interesting experiment that many pilots fail to notice. If you trim your aeroplane for straight and level flight then extend the gear the aeroplane will NOT slow down. Even though drag increases the nose will drop and the aeroplane will accelerate. Think through the whole explanation above until it makes sense to you why this happens.

In theory the same should be true for flap extension. Since flaps are always behind the CG their lift pitches the nose down and is the equivalent of trimming nose down. On some aeroplanes however, such as Cessna's, the deflected airflow from the flaps changes the angle of attack of the horizontal tail. On these aeroplanes the effect is like trimming nose up.

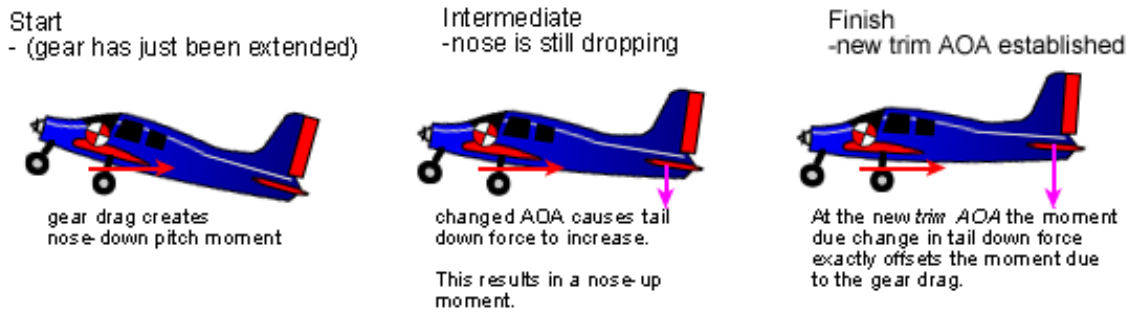
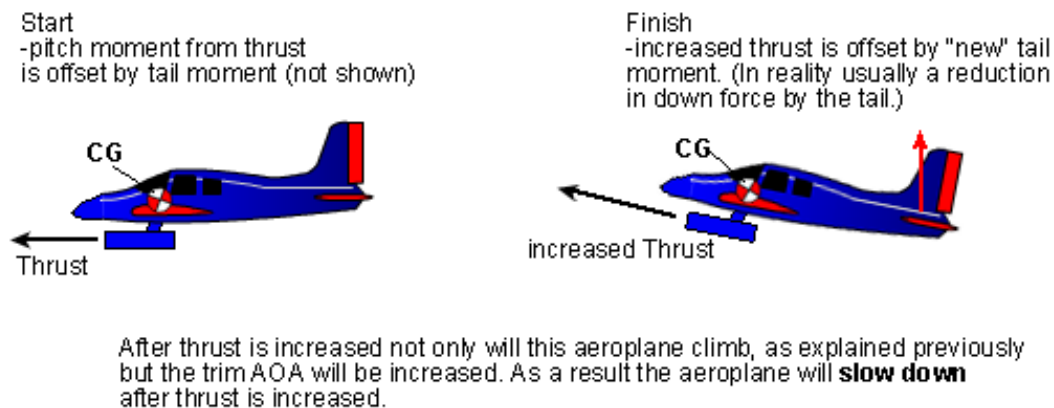


Figure 106

The bottom line is that when configuration is changed the trim angle of attack is changed. The aeroplane then finds itself in an **off trim** situation, which means that it finds itself not at the angle of attack it is trimmed for. It will then respond it must in order to get to the required trim angle of attack. The nose will go down (or up for flaps in a Cessna) and will usually overshoot the required angle of attack, then oscillate a few times before stabilizing at the new trim-angle-of-attack.

In order to prevent the oscillations described above pilots who don't want the trim angle of attack to change should quickly move the trim control after extending gear or flaps.



**Summary:** For aeroplanes with low thrust line, trim AOA increases with thrust.

Figure 107

Changes in power/thrust have the same effect as configuration changes. Figure 107 shows that on a particular aeroplane the thrust vector is below its CG. If thrust is increased it creates a nose up moment that brings the aeroplane into trim at a greater angle of attack. A reduction in thrust on this aeroplane would be the equivalent of trimming nose down.

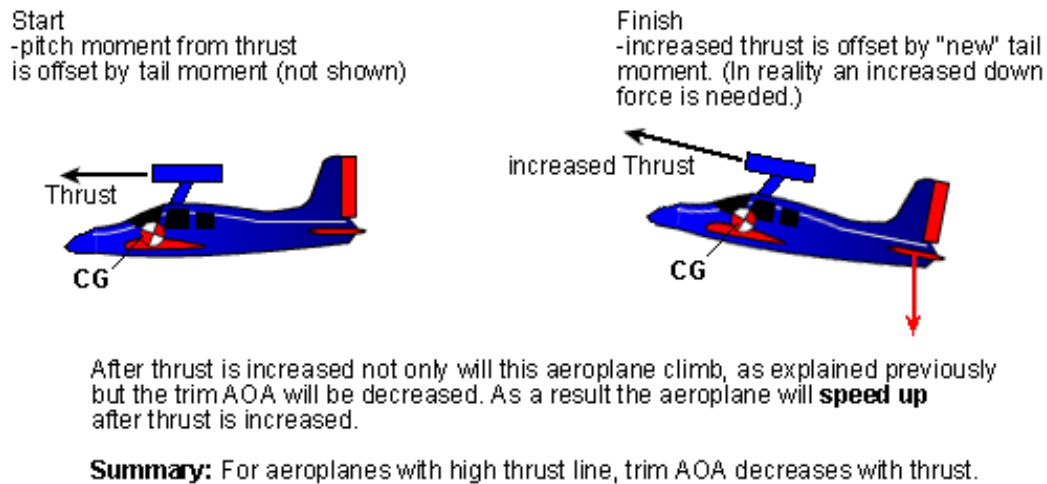


Figure 108

Figure 108 shows an aeroplane in which the thrust vector is well above the CG. If power is increased in this aeroplane it will pitch nose down until it comes back into trim at a lesser angle of attack. What would happen after that? We discuss that in the next section.

### Hero's Engine

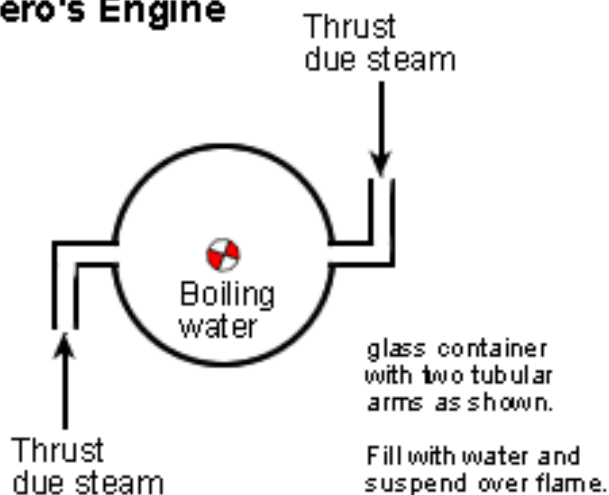


Figure 109

Figure 109 shows an ancient invention called Hero's engine. Steam boils in the center flask and exits through the tubes. As long as the water boils this engine will spin at great speed. This engine demonstrates rule 2. But, applying thrust on an aeroplane is quite different than Hero's engine. It does not cause unrelenting pitching motion; it simply changes the trim angle of attack. This is a vital point for pilots to grasp. Go over the above until it makes sense to you, because you cannot fly precisely and smoothly unless

you can trim an aeroplane effectively, and you cannot trim effectively unless you understand how small throttle movements change the trim angle of attack.

## ***Airspeed and Attitude Response to Power***

*This may be the most important topic in this book*, so I hope you are ready to wrap your mind around it.

Let's summarize all we have learned so far that is relevant to pitch/altitude control of aeroplane:

- In straight flight each angle of attack corresponds to one unique airspeed
- In a turn more angle of attack is needed to fly level at the same airspeed.
- In straight flight, at a particular airspeed, drag is the same whether the aeroplane is climbing, descending, or in level flight.
- Thrust is more than drag in a climb
- Thrust is less than drag in a descent
- Thrust is equal to drag in level flight
- Once trimmed an aeroplane maintains a constant angle of attack
  - Unless flaps, or gear are extended
  - Unless power is changed

We are now ready to analyze the mechanics of controlling an aeroplane. We will discuss the following scenarios. It is recommended that you actually try these out in the aeroplane you fly:

1. Trim the aeroplane for level flight in cruise. Pull back on the control column until the aeroplane loses 20 knots then let go of the control column. Use the rudder to keep wings level. What happens?
2. Trim the aeroplane for level flight at  $V_y$ . Smoothly apply full power – do NOT touch the control column. Use the rudder to keep wings level. What happens?
3. After completing 2, slowly reduce the power to idle. Keep wings level with rudder. What happens?
4. Trim the aeroplane for level flight well within the flap operating speed range. Extend  $10^\circ$  flaps. Do not touch the control column. What happens?

## **Scenario 1**

In case 1 every aeroplane will do approximately the same thing. The nose will drop and it will continue to drop until the airspeed reaches the trimmed cruise speed. At that moment the nose will stop dropping, but because the nose is down the speed will continue to rise. Once the speed is above cruise speed the nose will start to rise and will go higher and higher until the speed again drops to cruise speed. At that point the nose will stop rising,



and once the speed is below cruise speed again it will start to drop. This cycle will repeat several times. In almost all aeroplanes the cycles will damp out, which means reduce in size, until the aeroplane returns to level cruise flight. You will notice that you need right rudder when the nose is up and left rudder when the nose is down. The reason is explained later under P-factor.

When the airspeed is less than cruise angle of attack is more than trim, so the nose pitches down due to longitudinal stability. As soon as speed is more than cruise angle of attack is less than trim, so the nose rises due to longitudinal stability. Because power and configuration don't change in this scenario trim angle of attack does not change; so this scenario simply confirms the aeroplane's longitudinal stability.

### Scenario 2

All aeroplanes will wind up in a climb in this scenario because thrust will be more than drag. The interesting variation is whether airspeed increases or decreases.

If the thrust line of the aeroplane is below the CG, or the propeller slipstream passes over the horizontal tail increased power increases trim angle of attack. Consequently the aeroplane will *slow down*.

If the thrust line is above the CG increased thrust reduces the trim angle of attack. Consequently the aeroplane will *gain airspeed*.

In summary: All aeroplanes will enter a climb when power is added. Some slow down, and some speed up, depending on the thrust line position relative to the CG and horizontal tail.

### Scenario 3

All aeroplanes will enter a descent in this scenario, because thrust is becomes less than drag. Some aeroplanes speed up and others slow down. The logic is the same as in Scenario 2, can you think it through before reading what follows?

If the thrust line is below the CG or the propeller slipstream flows over the horizontal tail trim angle of attack will become less. The aeroplane will gain speed after power reduction.

If the thrust line is above the CG trim angle of attack will increase and the aeroplane will slow down.

### Scenario 4

This is the most interesting scenario because dramatically different things happen in different aeroplanes. It's a good idea to try this scenario in each new aeroplane you checkout in.

Many aeroplanes will pitch nose down, proving that flaps *reduce the trim angle of attack*. The aeroplane will pitch nose down until the new trim angle of attack is reached. Just as in scenario 1, it will overshoot and the nose will then rise. If you watch closely you can spot the speed at which each oscillation ends (up and down will end at the same speed.) After a few oscillations the aeroplane will stabilize at that speed, which will be *faster* than the previously trimmed speed. The aeroplane will be descending since drag has increased both due to the flaps and the higher speed. If the pilot wishes to prevent gain in airspeed it will definitely be necessary to trim nose up.

Some aeroplanes, such as single engine Cessnas, have flaps positioned so that the airflow deflected by the flaps changes the angle of attack of the horizontal tail. In this case extending the flaps *increases trim angle of attack*. As a result the nose pitches up. As explained above it will overshoot then oscillate and eventually establish flight at a new, lower speed. Flaps increase drag, but the aeroplane will have slowed down which reduces drag, so depending on how these two effects offset each other the aeroplane may climb, descend or fly level.

Hopefully the above discussion combined with some experimentation on your part will become part of your plan to learn to better control your aeroplane. The central point to note is that an aeroplane always seeks **equilibrium**, which means that it will continue to adjust its attitude until all pitch forces sum to zero.

Imagine you are on final approach for landing and are below the glidepath, what do you do?

If below glidepath on approach you must add power to reduce descent angle. But how will your airspeed respond. If you performed scenario 2 you know the answer. In many aeroplanes trim angle of attack increases, so you need to push forward to maintain speed. This is counter intuitive when you are low, but vitally necessary. If you have had problems with this in the past you now know why.

## Power Controls Climb, Trim Controls Airspeed

Previously we saw that thrust must be more than drag in a climb, less in a descent, and equal in level flight. Hopefully it is very clear that thrust (throttle) controls climb angle or rate, or vertical speed, whichever you prefer to say.

We have now seen that when we trim an aeroplane (elevator trim) we set it to fly at a certain angle of attack. if it were not for the complications described in the four scenarios above pilots would find that airspeed would remain largely unchanged following throttle changes. Broadly speaking, it would be the case that when power is increased the aeroplane would naturally enter a climb and slow down by a tiny amount, proportionate to the decrease in lift in a climb as opposed to level flight (review Figure 25.)

In a real aeroplane some trim change will be required when power is changed. If you conduct the experiments recommended above you will know what is required for your aeroplane.

Even with the complication caused by the thrust vector not passing through the CG it is broadly true that trim establishes airspeed. And it is definitely the case that an aeroplane is NOT trimmed to maintain level flight. When you adjust the trim while concentrating on maintaining altitude you are actually fine tuning airspeed until you establish a value where drag matches thrust. As a result, if you move the trim control faster than momentum permits the aeroplane to respond you will not be successful. This is the source of many inexperienced pilots' frustration at trimming.

An alternate technique is to get the aeroplane trimmed as close as you reasonably can to flying level then make "microscopic" throttle adjustments until the vertical speed settles on zero.

### ***CG Determines Stability***

Longitudinal and directional stability both depend on the moment created by the tail of the aeroplane. The longer the tail moment the more stable the aeroplane is. Therefore the farther forward the CG is the more stable the aeroplane.

As the CG is shifted back, the aeroplane becomes less stable, which means that the aeroplane resists changes in angle of attack less. At some CG location the aeroplane no longer resists changes in angle of attack. At that point we say the aeroplane is neutrally stable. More aft CG would make the aeroplane longitudinally unstable.

To be flyable an aeroplane must be statically stable. Even neutral stability will cause the pilot to over control and lose control within a few moments.

It is critical that pilots always load the aeroplane within the specified CG limits. These limits provide adequate stability. The CG limit is even more important than the maximum weight limit. If you load your aeroplane more than slightly beyond the aft CG limit you will almost certainly lose control – so don't do it!

A wing, as opposed to an aeroplane, is stable if the CG is ahead of the aerodynamic center (aerodynamic center was introduced in Figure 30.) Of course the CG is always well ahead of the stabilizers ac, so it is possible that the aft CG limit for the aeroplane as a whole may be further back than the ac of the wing. In such a case the wing would be unstable, but the stabilizer would overcome that instability so that the aeroplane as a whole remains stable. Even so, if the CG is allowed to move beyond the aft limit determined by the aircraft designer the aeroplane will become unstable.

## Air Density and Velocity Affect Stability Also

If an aeroplane is stable it will be more stable with more free stream dynamic pressure flowing past it. In other words aeroplanes are more stable at high velocity and in the more dense air at low altitudes (of course if the aeroplane is unstable it will be less stable in the same circumstances.) Speeding up and descending *won't make an unstable aeroplane stable*, but a properly loaded aeroplane will be more stable.

For the pilot of light aircraft this is seldom a factor but many jets have altitude limitations above which they cannot operate without yaw dampeners or other equipment operating due to reduced stability at high altitude.

## Conventional Tail

A conventional tail means an aeroplane that has a stabilizer consisting of a horizontal tail and elevators at the rear of the aeroplane. With a conventional tail the CG is always ahead of the stabilizer and the tail pulls down to keep the main wing producing lift. (Without the tail the main wing would “tuck down” to the zero lift angle of attack.) Net lift is the sum of the upward lift from the main wing and the downward lift from the stabilizer, this force must closely align with the CG varying from it only enough to offset any pitch moment created by the thrust and drag vectors. The wing is always forced to produce more lift than the weight of the aeroplane, which imposes an induced drag penalty commonly called **trim drag**. Note that the name is misleading because trim drag is not caused by trim tabs, it is inherent in the choice to have the stabilizer behind the main wing.

A stabilizer, being behind the CG, adds to the stability of an aeroplane. That is the main advantage of the configuration. If it is desired to make the aeroplane more stable it is a simple matter to increase the size of the stabilizer. The larger the horizontal tail the more longitudinal stability – just as a larger fin increases directional stability. This explains why it is called a *stabilizer*.

Since increasing the size of the stabilizer has few if any disadvantages it is normal to provide a stabilizer of sufficient size that the aeroplane can be stalled. In the discussion of Canard s below you will see that the opposite logic applies to them.

There is a greater nose down pitch moment when flaps are extended, thus a larger stabilizer is needed when flaps are installed. This is usually not a design problem, but once again the Canard is a different matter as we will soon see.

The main disadvantage of the conventional configuration is trim drag. This limitation can be overcome by the Canard design, which we turn to next.

## Canard

Figure 110 shows a Canard configuration. The fin is still at the back, but the horizontal tail and elevators are at the front (note that it is NOT called a stabilizer, as explained below.)

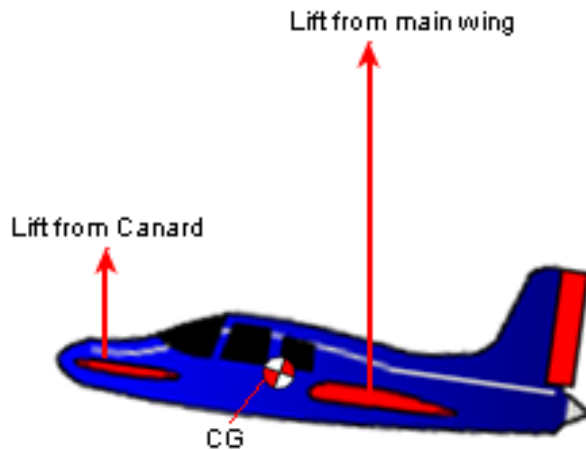


Figure 110

The CG must be well ahead of the main wing, but it is behind the Canard. A Canard keeps the main wing operating at a lift producing angle of attack by lifting up, rather than pushing down as a stabilizer does. The primary advantage of the Canard configuration is that it *reduces trim drag*. In other words the wing need produce lift of slightly less than the weight of the aeroplane with the Canard produces the rest of the lift. As a result there is less induced drag than with the conventional design.

Because the Canard is ahead of the CG it reduces the longitudinal stability of the aeroplane (It is in reality a *de-stabilizer*.) That is why a new name needed to be invented for it – hence Canard. The instability is easily overcome by setting a much more forward CG limit, thereby ensuring that the main wing is stable. If the aeroplane is redesigned so that the Canard is made bigger stability is reduced. Be sure to fully think this point and make sure you can see why.

On any aeroplane (Canard or conventional) increasing the size of the horizontal tail provides more controllability. But with a conventional design it also makes the aeroplane more stable. With a Canard increased controllability conflicts with stability. So if the aeroplane had a large Canard as would be needed to do short field takeoffs or pitch into a stall it might well be unstable. The need to ensure stability and safe stall characteristics necessitates that all Canard aeroplanes be designed with small Canards, and thus be unable to stall and generally have poor short field takeoff and landing performance.

Flaps are a problem with Canard designs. We learned previously that extending flaps pitches the nose down, so the Canard would need to be large enough to offset this nose down tendency. If a Canard is large enough to bring the aeroplane close to a stall with flaps extended it is large enough to fully stall the aeroplane without flaps. That is a serious problem for Canard aeroplanes. With a conventional configuration the loss of lift from the wing at stall produces a natural nose down pitch moment that tends to promote stall recovery. But with a Canard, loss of lift from the wing, with the Canard lifting up, causes the aeroplane to pitch up further into a deep stall that may be unrecoverable.

Fortunately there is a fairly simple solution; design the Canard so that it always stalls before the wing. That explains why many Canard aeroplanes do not have flaps, and those that do require complex mechanisms to change the “stall speed” of the Canard when flap setting is changed.

Two disadvantages of Canards are that they cannot have flaps without complex methods of changing the size or effectiveness of the Canard when flap settings are changed. And, because of the flap limitation combined with the need to keep the Canard small to optimize stability they are generally unable to reach full  $C_{Lmax}$ . Consequently Canards are not good short field aeroplanes.

### Hybrid Configuration

Figure 111 shows a hybrid (mixed) configuration. This aeroplane has both a Canard and a conventional tail. The design has a fixed Canard, i.e. one that has no elevators, which provides up force during cruise, so the stabilizer does not need to make a down force. This reduces trim drag in cruise. But, because the conventional tail is behind the CG it can be as large as desired so good takeoff and landing performance can be maintained.

*The fixed Canard must be installed at a greater angle of attack than the main wing so that it stalls first. This ensures a safe nose down moment in the event of stall.*

#### Hybrid Configuration

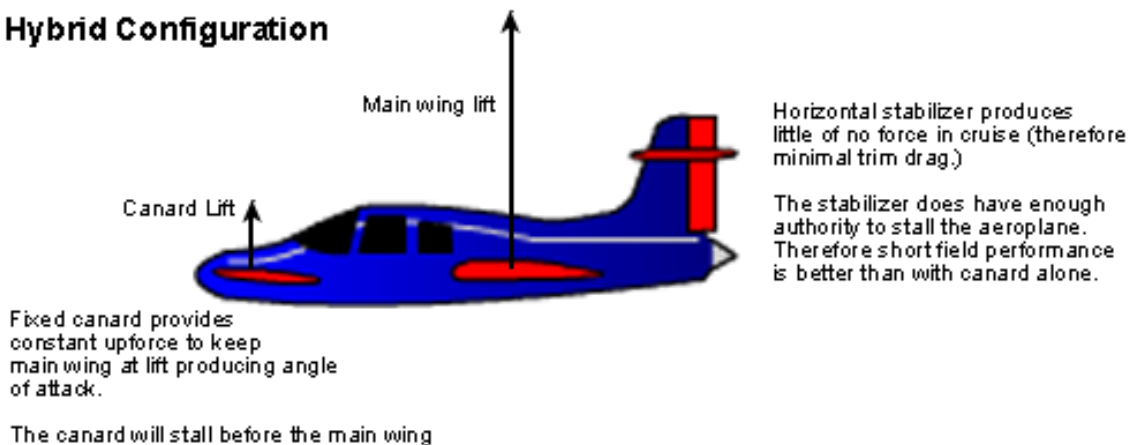
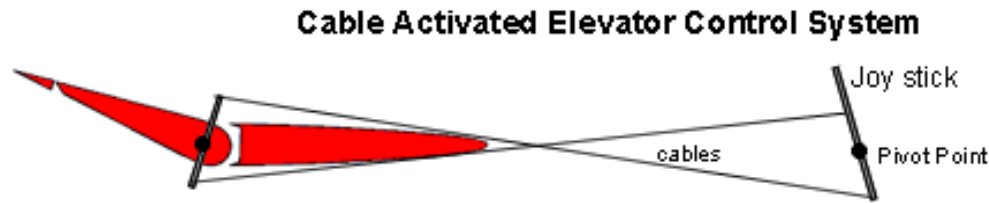


Figure 111

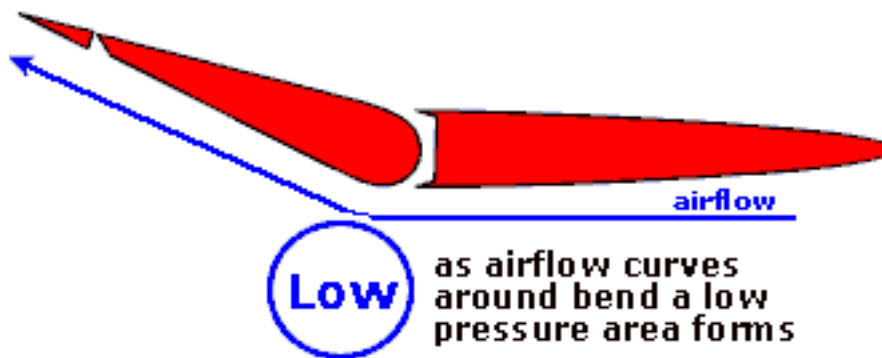
### Stick Force and Artificial Feel

Figure 112 shows a simple cable control system. The pilot moves the elevator, rudder, or aileron using muscle power.



**Figure 112**

Figure 113 shows that a change in camber is caused by a deflected control, which alters the static pressure. The pressure difference always acts to bring the control back to the center (i.e. neutral, or zero deflection) position. The pilot must apply a force to the wheel to hold the desired control deflection against this static pressure field. This explains the resistance pilots feel to any deflection of the controls in flight.



**Figure 113**

You have already learned that a trim tab can be used to hold the control deflection.

The amount of force a pilot feels depends on the extent of control deflection, the speed of the aeroplane, and the density of the air.

If an aeroplane is more stable then more control deflection is needed to change angle of attack a given amount. Thus the elevator controls will feel heavy at forward CG and be more effective and feel lighter as CG moves aft.

### **Static Control Balancing**

Figure 114 shows a typical elevator control and points out that it, like everything else in the world, has a CG. The momentum of an object acts at its CG.

As explained above, a control surface tends to move back to the neutral position, but because it has momentum it will overshoot and then tend to oscillate back and forth. The result can be rapid oscillations of the control, very much like a flag “fluttering” in the

wind. Indeed, aeronautical engineers call this phenomenon flutter, and if severe it can “rip” a control off an aeroplane (not a good thing.)

To prevent flutter controls are **statically balanced**. Figure 115 shows how a weight is added to move the CG of the control into alignment with the hinge line. In reality it is seldom necessary to move the control’s CG all the way to the hinge line, but some balancing is needed on all “free floating” control surfaces to guard against flutter. Pilots should be aware that flutter is a harmonic phenomenon that peaks at certain dynamic pressures (speed and density altitude.) While any certified aeroplane will be free of flutter when flown as designed damage to a control or operation outside the certified speed range can excite flutter.

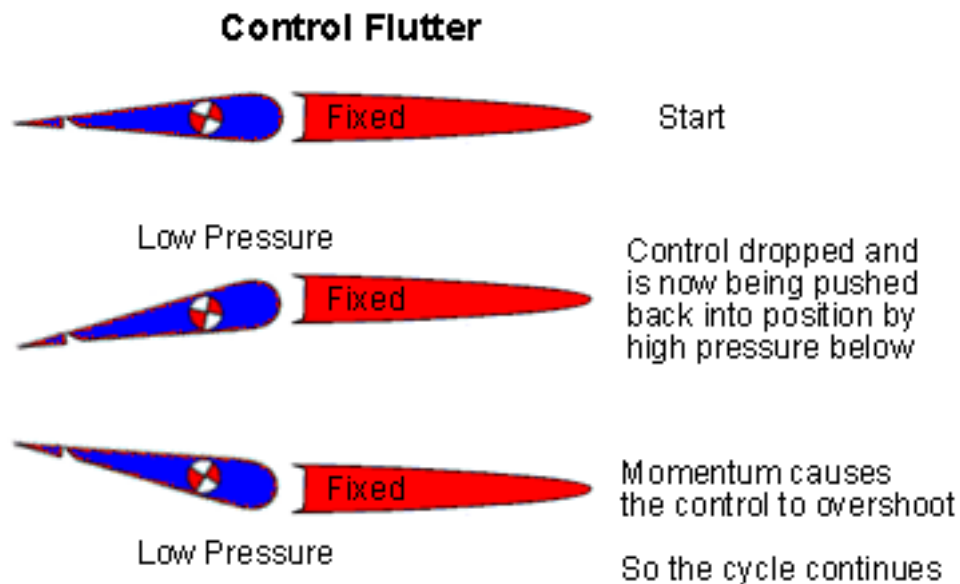


Figure 114



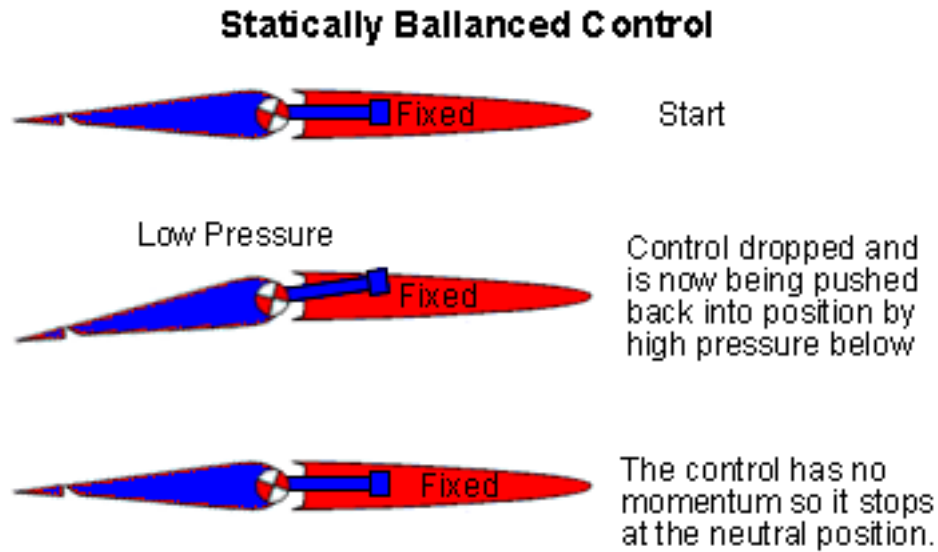


Figure 115

### Dynamic Control Balancing

Figure 116 shows a control surface strategically shaped so that a portion of it produces a moment that helps the pilot move the control. This is commonly called a control **horn**, or a **dynamic balance**. Dynamic balance provides more assistance at high speed, which is just what the pilot needs. They are an easy way to make the controls lighter and easier for the pilot to move.

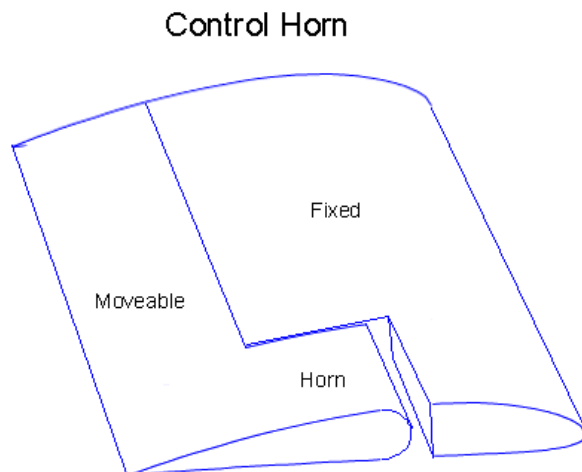


Figure 116

### ***Artificial Feel***

As noted above, it takes force to move a control surface against the static pressure field. In larger aeroplanes the amount of force is too great for the pilot. In this case a power assist feature is added, not unlike power steering in a car.

Depending on how the powered controls operate there the pilot may be able to change angle of attack without feeling any force on the column. In such a case the pilot will be very prone to over control. In fact it will be almost like flying an aeroplane with no stability. **Artificial feel** is needed. This could be as simple as a bungee system on the control column so the pilot feels some force, but is more likely a complex system of electronics or hydraulics that vary the control column force depending on airspeed and control deflection realistically simulating what a cable based control system would feel like.

### ***Artificial Stability***

Artificial stability is very different than artificial feel. In the artificial feel system described above it was assumed that the aeroplane is stable, it was only necessary to make the controls feel “normal” so the pilot doesn’t over control.

We noted earlier that a trim drag penalty ensues from the need to keep the CG forward. If the CG is moved aft an aeroplane becomes less stable, but there is less drag, and it becomes more easily maneuverable. It is possible to turn control over to a computer that can fly an aeroplane even if it is unstable. The benefit is improved fuel efficiency; the negative side is that the aeroplane would be un-flyable without the computer.

To date only military aeroplanes have been built with artificial stability.

### ***Adverse Yaw / Aileron Drag***

Ailerons produce roll by increasing the camber of one wing and reducing camber on the other wing.

We already know that when lift increases induced drag increases. Unfortunately that means the wing being lifted also experiences more drag, which tends to yaw the aeroplane opposite to the direction of the desired turn. Since this is adverse to the desired turn it is called **adverse yaw**; because it is caused by the ailerons it is also known as **aileron drag** (the two terms are synonymous.)

Adverse yaw must be compensated for. The pilot must apply a small amount of rudder in the direction of the desired turn just as the ailerons are applied. Remember that once the turn is established ailerons are returned to neutral, so there is no longer any adverse yaw, therefore the pilot should release the rudder once the turn is established. Rudder is required again when the turn is stopped. The process of compensating for aileron drag is commonly called *coordinating a turn*. Pilots learn to judge the “quality” of their turn coordination by checking that the ball in the turn coordinator remains centered during a turn.

## ***P-Factor***

A turning propeller is really just a rotating wing; it produces lift because it has an angle of attack and velocity.

Figure 117 shows that as an aeroplane's angle of attack increases the down going propeller blade experiences an increased angle of attack while the up going propeller blade has a reduced angle of attack. As a result the thrust is no longer exactly centered on the propeller hub – it is offset to the right. Pilots must anticipate that when angle of attack increases the aeroplane will yaw a bit to the left, so right rudder will be needed to prevent a slip. Clearly this will apply to all climbs, but also applies anytime the aeroplane is flying at less than cruise speed, even in level flight.

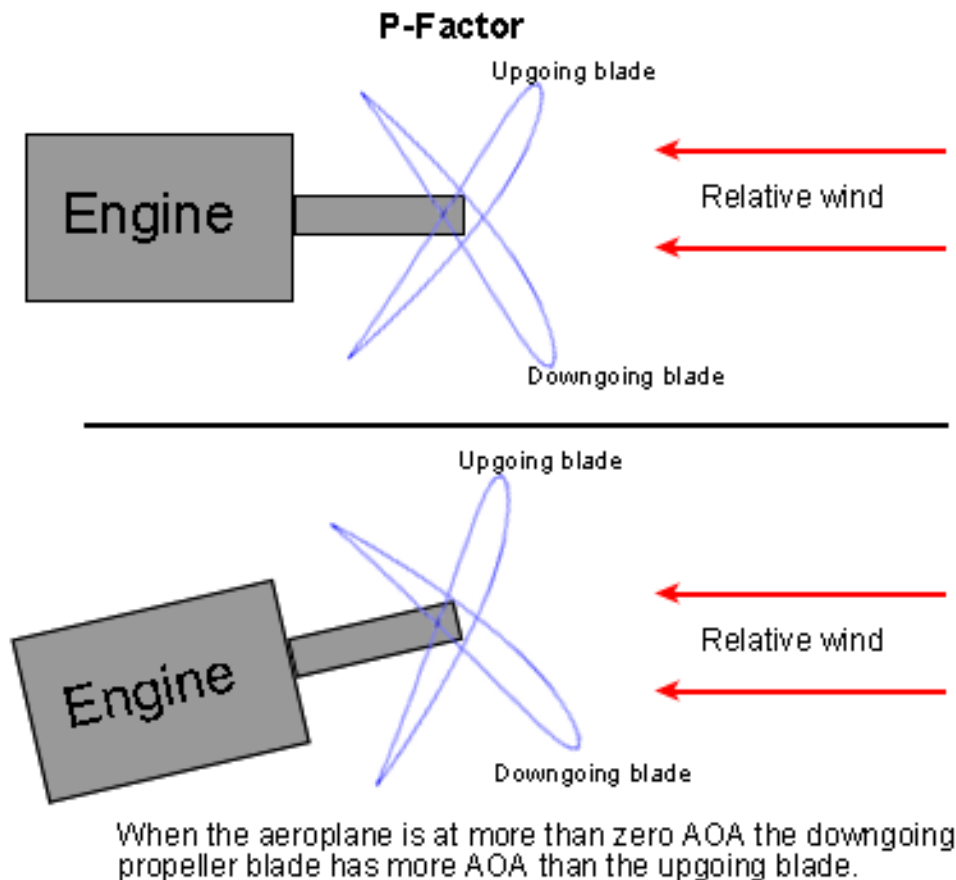
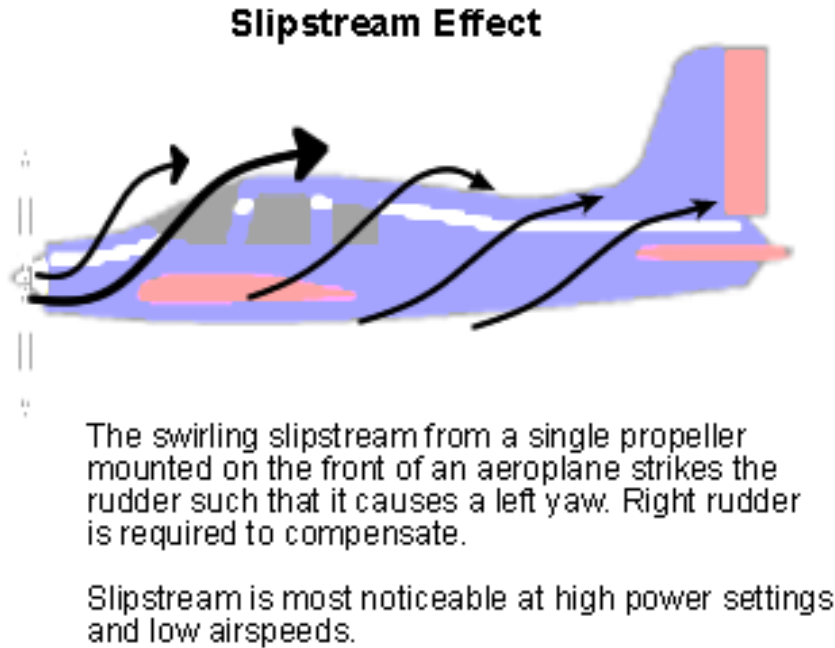


Figure 117

## ***Slipstream Effect***

The slipstream from the propeller spirals around the fuselage, as shown in Figure 118. The airflow impacts parts near the rear of the aeroplane at an angle of attack causing yaw. In most cases this adds to the P-factor (if the fin protrudes downward rather than up then it offsets P-factor.)



**Figure 118**

Aeroplanes must be designed to “allow” for the amount of slipstream effect in cruise. This is usually done by slightly offsetting the fin. It can also be done by slightly offsetting the thrust line, or a bit of both. As a result most single engine propeller aeroplanes yaw right, requiring left rudder, whenever power is reduced below cruise setting. This generally means that left rudder is needed in a descent with low power.

### ***Gyroscopic Effect***

The rotating propeller and crankshaft in the engine act like a gyroscope. In a modern aeroplane where the engine rotates as per Figure 119 the gyroscopic effect is such that when the nose is pitched up it tends to yaw the aeroplane right, and vice versa. Note that this effect works opposite to P-factor.

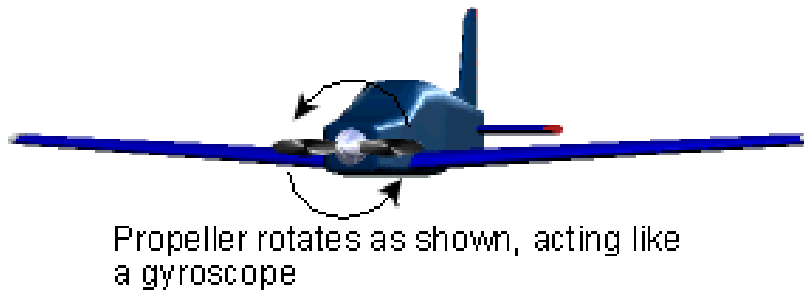
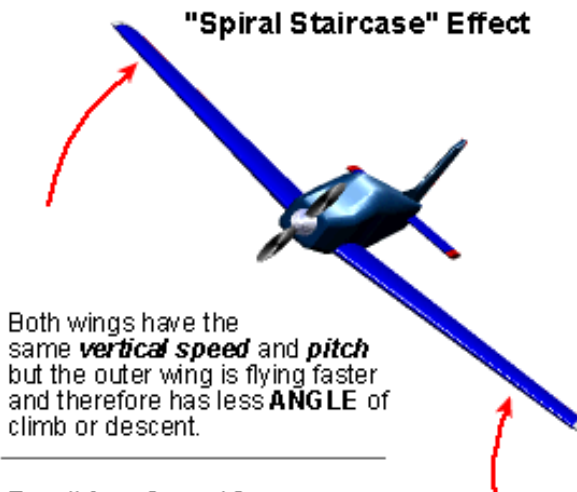


Figure 119

In addition, on the takeoff roll the rotation of the engine and propeller tends to “twists” the aeroplane slightly so that more weight rests on the left main wheel. This increases drag and causes the aeroplane to veer left on takeoff. Pilots notice that right rudder is needed to keep straight during the takeoff roll.

*Note that if the engine rotates opposite the direction shown in the figures P-factor, slipstream, and gyroscopic effects are reversed from that explained above.*

### **Spiral Staircase Effect**



Recall from figure 16:

$$\text{Pitch} = \alpha + c$$

$$\alpha = \text{Pitch} - c$$

Angle of attack ( $\alpha$ ) on the wings is reduced by  $c$  in climbs but increased by  $c$ , which is negative, in descents.

Therefore the outer wing has the greatest angle of attack in a climb, but the inner wing has the greatest angle of attack in a descent.

Figure 120

Imagine yourself walking up or down a spiral staircase with one hand on each handrail. Obviously your outer hand must slide along the handrail faster than your inner hand. Yet both start up (or down) the stairs at the same time and reach the next floor at the same time so they both have the same vertical speed. The slower hand must travel at a greater angle of climb or descent.

In the case of a spiral staircase you can visually see that the inner handrail is much steeper than the outer handrail.

When an aeroplane climbs or descends while turning the inner wing must climb or descend at a greater angle than the outer wing. We designate that angle as  $c$ , and it is positive for climbs and negative for descents. Refer back to Figure 18 in which we defined the relationship between pitch, angle of attack and climb. The relationship is:

$$\text{Pitch} = \alpha + c$$

We now solve this equation for  $\alpha$ :

$$\alpha = \text{Pitch} - c \quad [\text{both wings have the same pitch}]$$

The inner wing's angle of attack is reduced more in a climb, since its angle of climb is greatest. In other words, the outer wing has a greater angle of attack in a climb.

In a descent  $c$  is negative, so the inner wing's angle of attack is increased more than the outer wing. The inner wing has the greatest angle of attack in a descent.

As a result of the spiral-staircase-effect aeroplanes become laterally unstable in climbing turns. This is because; in a climbing turn the outer wing has both greater speed and greater angle of attack and thus produces more lift than the inner wing. Consequently the aeroplane tends to roll into the turn. The pilot must prevent this by applying a small amount of aileron opposite to the turn. Pilots should also note the deflected ailerons produce a small amount of aileron drag and thus require a small amount of coordination during climbing turns.

In a descent the inner wing has the greatest angle of attack, but it has the lowest speed. This tends to offset the effect explained previously in Figure 99, as a result the aeroplane becomes *more* laterally stable (less unstable?) in a descending turn.

Many VFR pilots don't notice the effects described above, but they are a factor during instrument flight. Pilots should know that if they are distracted from controlling the aeroplane in a climbing turn (say while performing a post takeoff checklist) the aeroplane will tend to roll further into the turn and control may be lost. But the same distraction during a descending turn (say while performing the pre-landing checklist) will usually result in the aeroplane continuing stably in the turn, or slightly rolling out.

All pilots should experiment with the effects described above to confirm them for themselves. To do so, simply trim (elevator trim only – leave aileron trim neutral) the aeroplane in a climb and release the control wheel while keeping the ball centered with the rudder; the aeroplane will roll ever steeper. Next trim the aeroplane in a moderate bank descending turn and release the controls – the aeroplane will either remain stably in the descending turn, or may roll out of the turn.

### ***Stall and Spin Recovery***

The spiral staircase effect is a factor in spin entries. We will return to that in a moment.

We learned earlier that a wing always stalls at a certain angle of attack. Once that angle of attack is exceeded the wing is stalled. If both wings stall simultaneously the result is usually a nose down pitch change and a steep descent as shown in Figure 121. Even though the nose drops, angle of attack may continue to exceed the critical angle (stall angle of attack, refer to Figure 31.) If the pilot does nothing to recover or worse holds the aeroplane in the stall, a steep descent due to the high drag will ensue. This is depicted in the third stage of Figure 121. Recovery from a stall is simple; the pilot must push forward on the control column thereby reducing angle of attack. The aeroplane can then be recovered from the ensuing dive, with the pilot taking care not to exceed the critical angle of attack again.

Many aeroplanes will recover from a stall if the pilot simply releases the control column. The aeroplane naturally returns to its trimmed angle of attack, which presumably is less than the stall angle of attack. A very few aeroplane suffer what is known as a **deep stall**. In a deep stall something happens that forces the aeroplane to remain stalled. The most common problem is that disturbed air from the main wing flows over the tail creating a nose up pitch moment. If this effect is strong enough the aeroplane may remain in the stall and be difficult to recover. Poorly designed Canard aeroplanes suffer deep stall problems. But, as previously explained a well designed Canard is not able to stall (i.e. the Canard is not large enough to lift the nose of the aeroplane to the stalling angle of attack.) But, if you miss-load a Canard it may be possible to stall, and subsequently encounter a deep and unrecoverable situation. In the event of a deep stall the pilot should apply full nose down elevator and apply power. If this does not result in recovery the pilot should attempt to roll the aeroplane, with rudder, into a banked attitude. Once banked the aeroplane's directional stability should takeover and bring the nose down, however a large bank angle may be needed.

Stalls are very dangerous if encountered close to the ground. They should be practiced only at a safe altitude. In normal flight operations the pilot must make every effort to prevent a stall by remaining well above the stall speed. It is important to remember that stall speed increases in a turn; therefore more speed is needed when turning. This is usually not a problem, but one scenario that continues to lead to stall accidents is attempting to turn back to the runway following a low altitude engine failure after takeoff. This maneuver should only be attempted if airspeed can be kept up to the normal

approach speed during the glide, which for most aeroplanes means about 1.3 to 1.4 times the published stall speed. This provides adequate margin to perform a 45° bank turn.

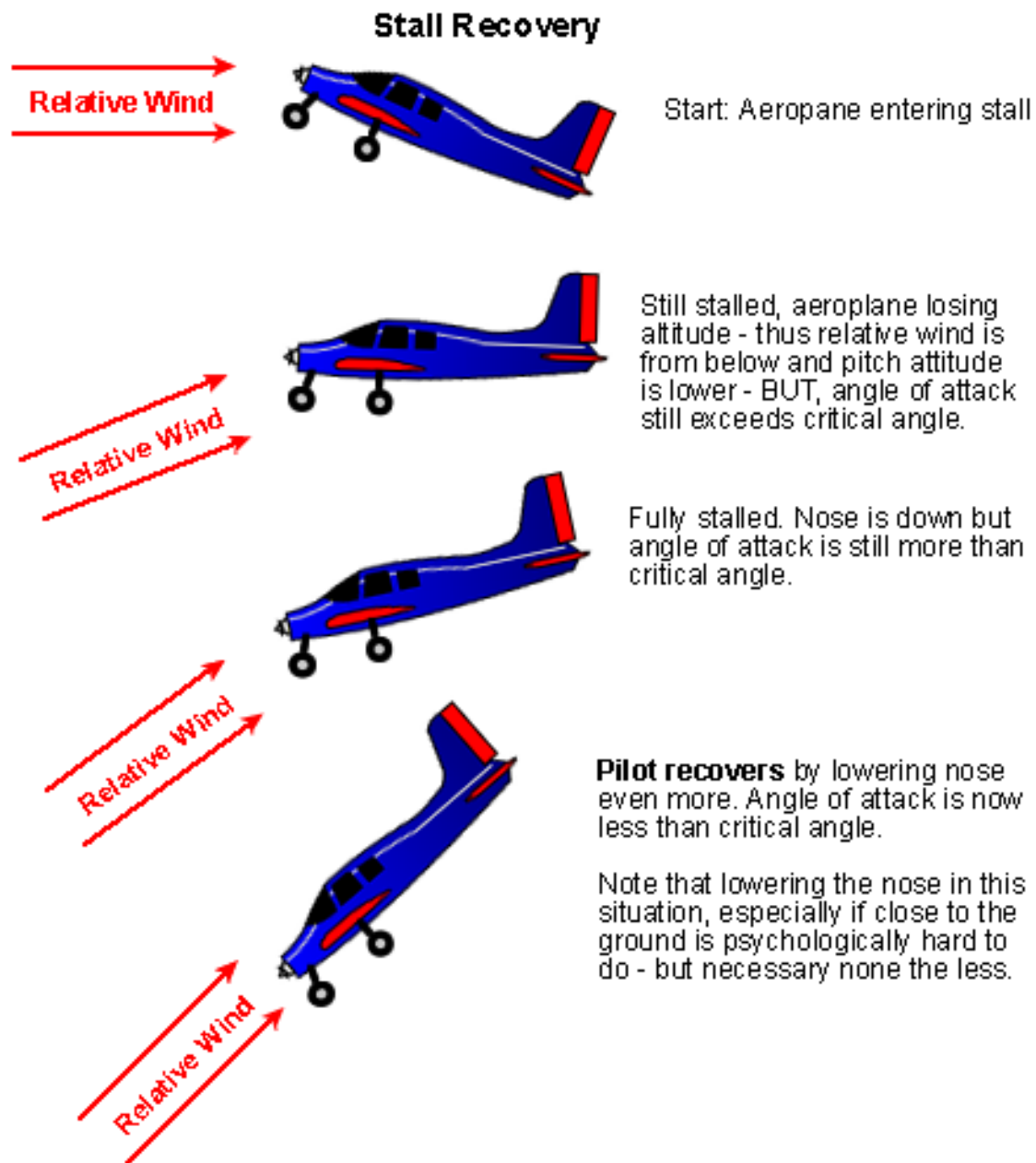


Figure 121

Aircraft are always equipped with some sort of stall warning device to prompt the pilot to recover before stalling. In addition an aerodynamic buffet often precedes the stall. The buffet is caused by the flow separating from the top of the wing and striking the tail. Unfortunately neither of these warnings is very reliable in light aircraft. Stall warning devices often fail. Or they may give so much warning (i.e. activate well before the stall) that pilots become complacent and learn to ignore them. Buffet is not a reliable indicator



of impending stall in many airplanes. T-tail designs usually don't experience it at all and many other designs experience buffet such a short time before the stall that the pilot doesn't have time to react. Thus pilots must learn to avoid situation that could lead to a stall and avoid them.

If one wing is at a greater angle of attack than the other at the time the aeroplane stalls a spin will result. The most common reason why one wing would have a different angle of attack is the spiral staircase effect, discussed previously. A pilot can induce a spin by applying rudder just prior to stalling; the resulting yaw slows the inner wing, increasing its angle of attack if the aeroplane is descending.

In a stall, if one wing has greater angle of attack it produces more drag (as always) but also produces less lift. (Refer back to Figure 31 and note that when angle of attack exceeds that for stall lift decreases with further increases in angle of attack.) The situation is shown in Figure 122.

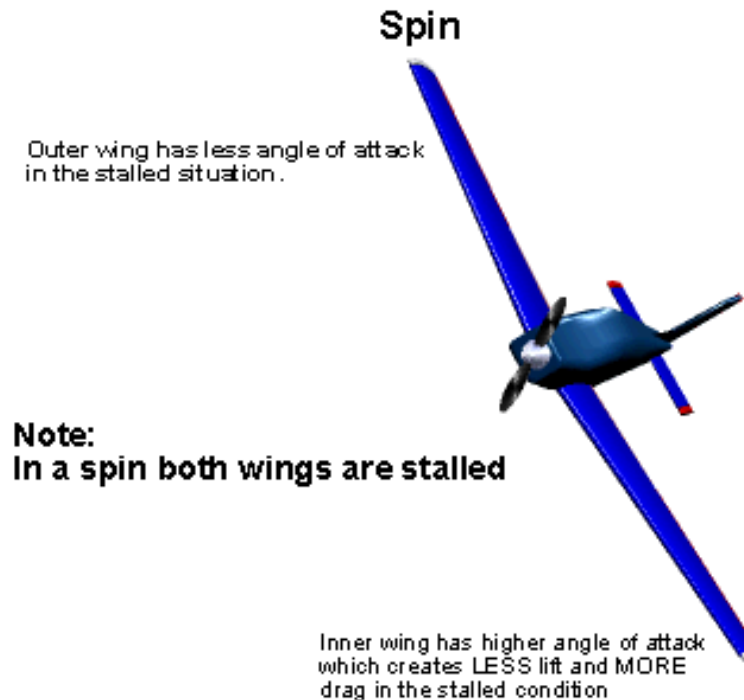


Figure 122

Since it is always the inner wing that has the greatest angle of attack in a descent (review spiral staircase effect) the inner wing produces the least lift and the most drag. That causes the aeroplane to continue to roll toward the lower wing and simultaneously yaw toward the lower wing. The result is a self-sustaining rolling and yawing motion called **auto-rotation** or a **spin**. All spins involve a continuous rolling and yawing motion. Spin attitude can vary considerable. In light aircraft a fairly nose down attitudes are quite

common. Nose down spins usually have quite high roll rates. Many larger aeroplanes tend to spin “flat” which means with a less nose down attitude, and usually a higher yaw rate. During a spin power, elevator, and aileron inputs interact to affect the spin attitude and provoke oscillations in the attitude.

Increasing propeller rpm during a spin increases the yaw rate. The magnitude of the effect depends upon the size and weight of the prop. Pitch attitude response to change in power depends on the direction of the spin, the angle of attack, and the ratio of roll to yaw in the spin. In a left spin adding power causes both faster and flatter spinning. In a right spin, adding power produces a faster spin, but with less noticeable pitch change.

Applying aileron opposite the direction of a spin levels the wings and flattens it while dampening roll and pitch oscillations. Applying aileron in the direction of a spin steepens the attitude and amplifies roll and pitch oscillations.

To recover from a spin the pilot must stop the yaw and reduce the angle of attack. The amount of forward elevator required for a spin recovery depends on the effectiveness of the rudder. When opposite rudder is capable of significantly slowing the rotation then the elevator typically only has to move forward to its neutral position. If, on the other hand, the opposite rudder has minimal effect on the spin, then the elevator may have to move all the way to the forward control limit.

The spin recovery technique specified in the POH should always be followed. However, if none is provided the following procedure will work in almost all cases:

1. Power--Off.
2. Ailerons--Neutralize (& Flaps "up").
3. Rudder--Apply fully opposite to the direction of yaw.
4. Elevator--Push forward beyond neutral.

Hold inputs until rotation stops and then neutralize the rudder and ease out of the dive.

The steps in the recovery process should be followed sequentially i.e. one, then two, etc. Do not try to do them all at once but, there should be no delay between them either.

Some aeroplanes will not recover from a spin. This is especially likely if the airflow over the elevators or rudder is disturbed making them less effective. Just as with the deep stall described above, this is called a **deep spin**. To recover from a deep spin may not be possible. The pilot should lower the undercarriage; try full power, and sustained application of full rudder and nose down elevator. If that does not work then desperate measures would involve opening doors and windows in an attempt to disrupt the auto-rotation. As a final resort try extending flaps (normally extending flaps will make the situation worse, but if all else has failed little will be lost by trying.)

Pilots may be interested to note that test aeroplanes are fitted with “spin chutes” during certification testing. The test pilot can deploy this parachute if the aeroplane gets “stuck” in a spin. The small chute deploys from the tail of the aeroplane and creates enough drag to bring the aeroplane out of the spin. No production aeroplanes have spin chutes however, so pilots should make every effort to avoid spins. The same advice applies as for stalls, with the additional admonition to avoid “heavy footed” use of rudder at low airspeeds. If the aeroplane is skidding at the time it stalls a spin will almost certainly ensue.

## Multiengine Aerodynamics

Riddle: Why does an aeroplane have two engines?

Answer: because it can't fly on one. (big smile ;)

*Historical Note: In the early 1960s when the first jet transport aircraft were built they had four engines (B707 and DC8.) The reason was primarily that no engine manufacturer could deliver two engines with enough thrust. For the same reason a B52 bomber had eight engines. Modern jet airliners have only two engines, which is all that redundancy requires.*

On light piston powered twins the second engine adds lots of extra power, which makes most twins climb very well, and gives them quite high service ceiling. But if an engine fails the performance of the aeroplane is often so poor as to be almost (or actually) dangerous. Transport Canada regulations don't require a twin with a gross weight less than 6000 lb to be able to climb on one engine (and many won't.)

Transport category aeroplanes, certified under CAR 525 are required to be able to continue a takeoff after a specified speed, called  $V_1$ . They are able to takeoff and fly an IFR departure and then an IFR approach all with one engine out. But, don't count on being able to do that in a piston engine aeroplane.

*NOTE: In what follows I have assumed an aeroplane with two engines. Therefore, when I speak of "single engine" that means that one of the two engines has failed.*

*I have also assumed that the engines are mounted bi-laterally. The special case of engines mounted inline is NOT covered.*

### **Climb Performance with One Engine Out**

Climb performance depends on excess power, that is to say the amount of extra power available beyond what is required to fly level.

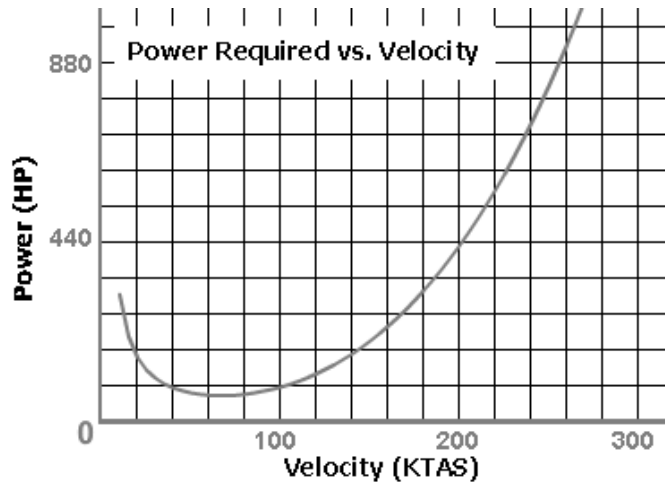


Figure 123

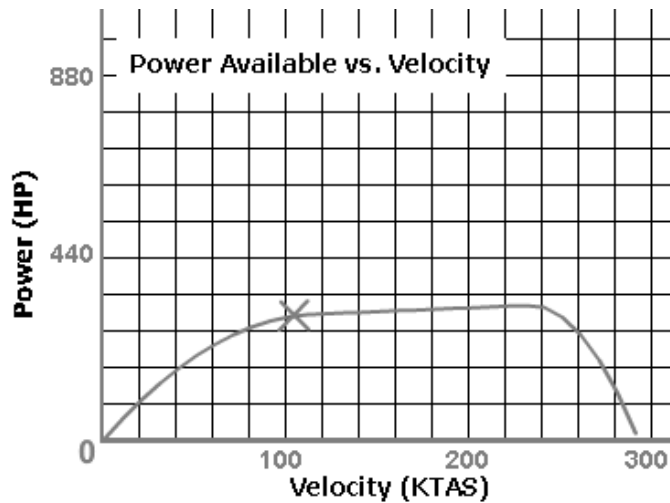


Figure 124

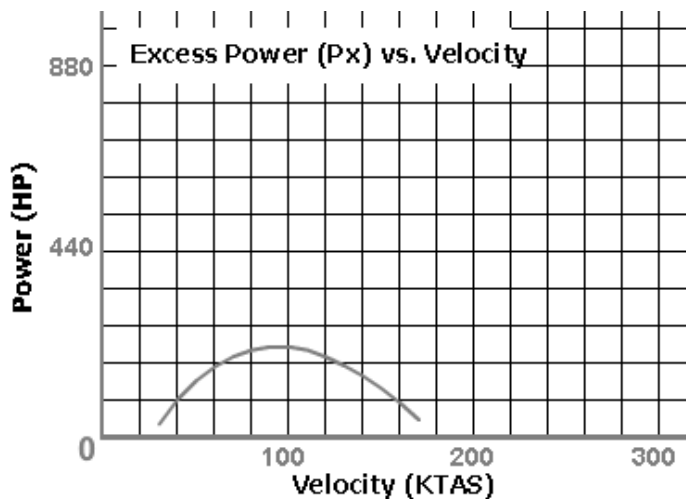


Figure 125

Figure 123 shows typical power required for level flight (Pr.)

For this particular aeroplane 440 HP is needed to fly 200 KTAS. But, only 88 HP is needed to fly 100 KTAS.

Figure 124 shows power available (Pa.) This is the amount of engine power (BHP) the propellers can turn into useable power (Pa.)

Pa for this aeroplane is roughly 290 horsepower from 100 to 240 KTAS. Above and below those speeds the power available drops off (due to poor propeller efficiency.)

Note: the "X" represents the current speed and power setting.

If you subtract power required from power available the difference is excess power (Px) as shown Figure 125.

For this particular aeroplane the maximum value of Px is about 200 HP and that occurs at 95 Knots.

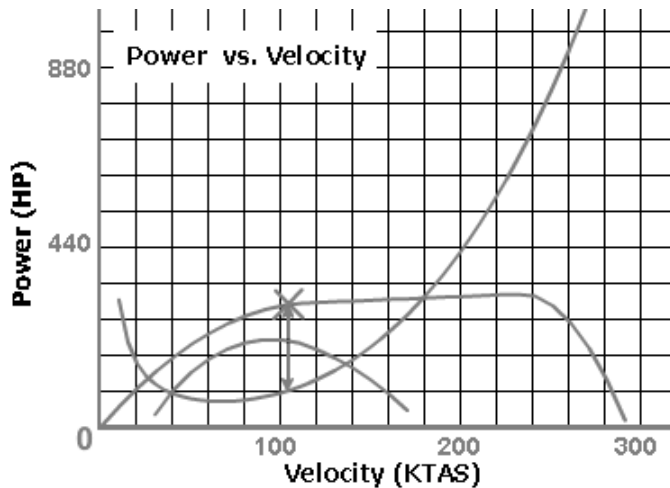


Figure 126

Figure 126 is simply all three of the above diagrams superimposed one over the other.

The “X” marks the current airspeed and power. The two-headed vertical line represents excess power, i.e. Power available minus power required, at the example speed of 105 knots.

Using math symbols we write:

$$P_x = P_a - P_r$$

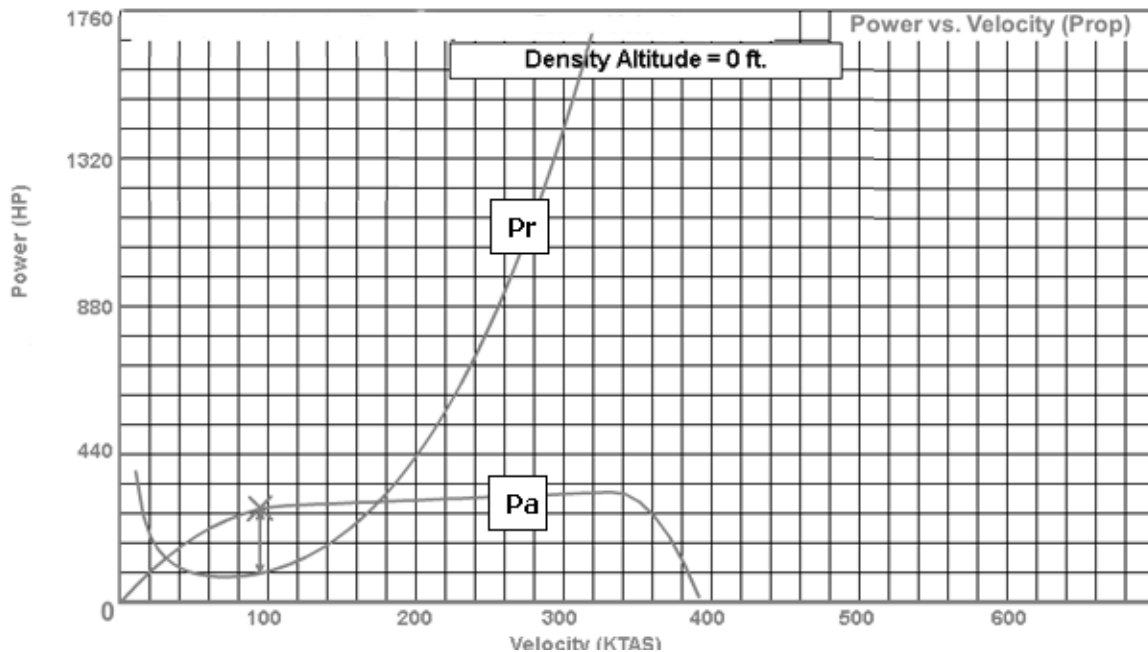
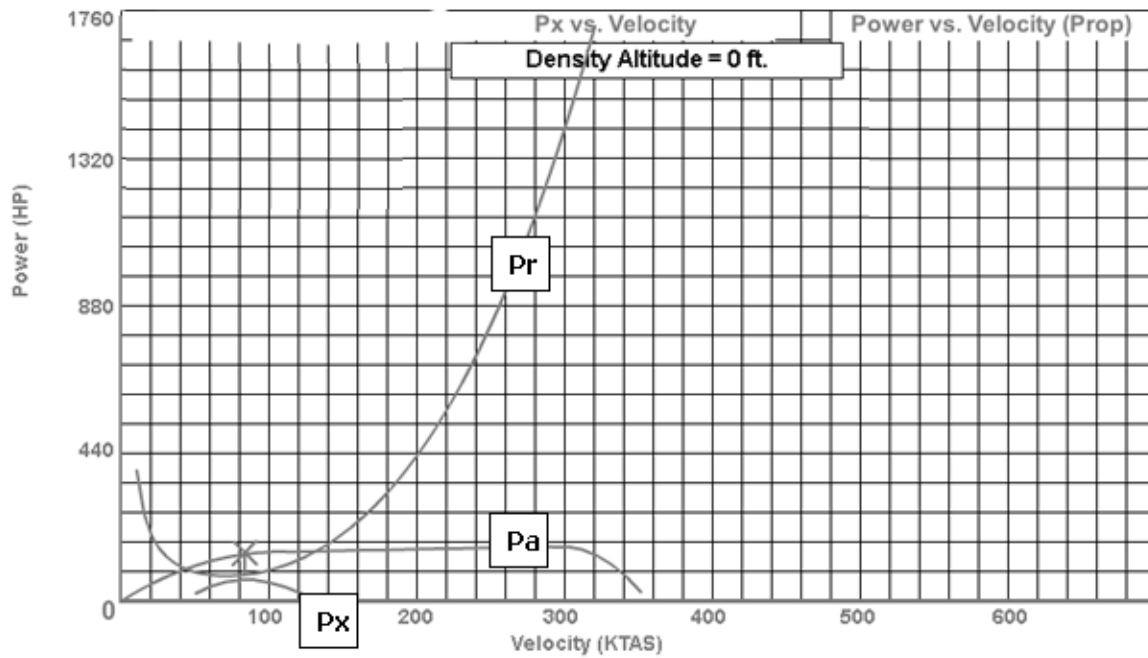


Figure 127

Figure 127 shows the approximate  $P_r$  and  $P_a$  curves fore a Travelair at sea level. Both engines are producing full power. If you recall the rate of climb equation developed earlier:

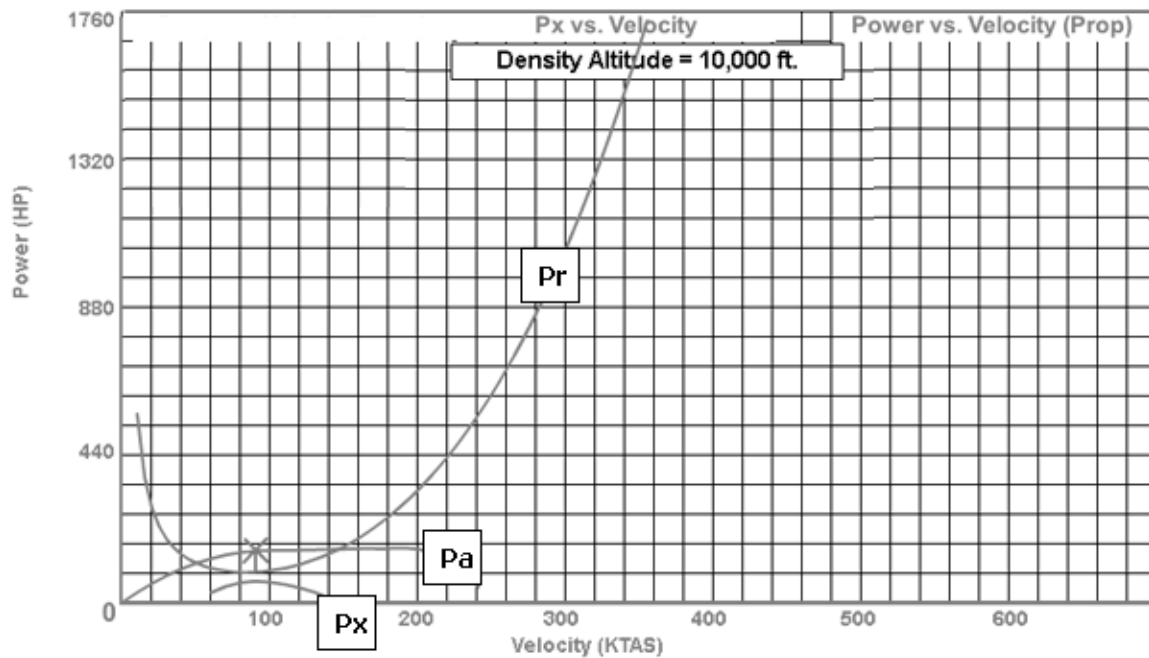
$$\text{Rate of climb} = P_x \times 33,000 / W$$

With two engines there is lots of  $P_x$ , so the aeroplane can climb very rapidly – more than 1000 fpm. Now imagine how the graph changes if  $P_a$  is cut in half.



**Figure 128**

In Figure 128  $P_r$  is the same as before, but the  $P_a$  is cut in half. The “gap” between the curves is  $P_x$ . You can see that there is very little  $P_x$ . You must remember that when one engine quits climb rate is substantially cut, because ALL the power lost comes out of  $P_x$ . Now think about what would happen if the engine quit when flying at a higher altitude.



**Figure 129**

Figure 129 shows  $P_x$  with two engines as the aeroplane climbs through 10,000 feet. Notice that  $P_a$  is less than at sea level because the engines produce less power in the thin air at 10,000 feet. NOTICE also that power required is slightly greater because the aeroplane must fly faster in the thin air (remember that power equals thrust time velocity.) In this case there is enough power for the aeroplane to climb at about 550 fpm on two engines at 10,000 feet. Now let's see what happens when one engine quits.



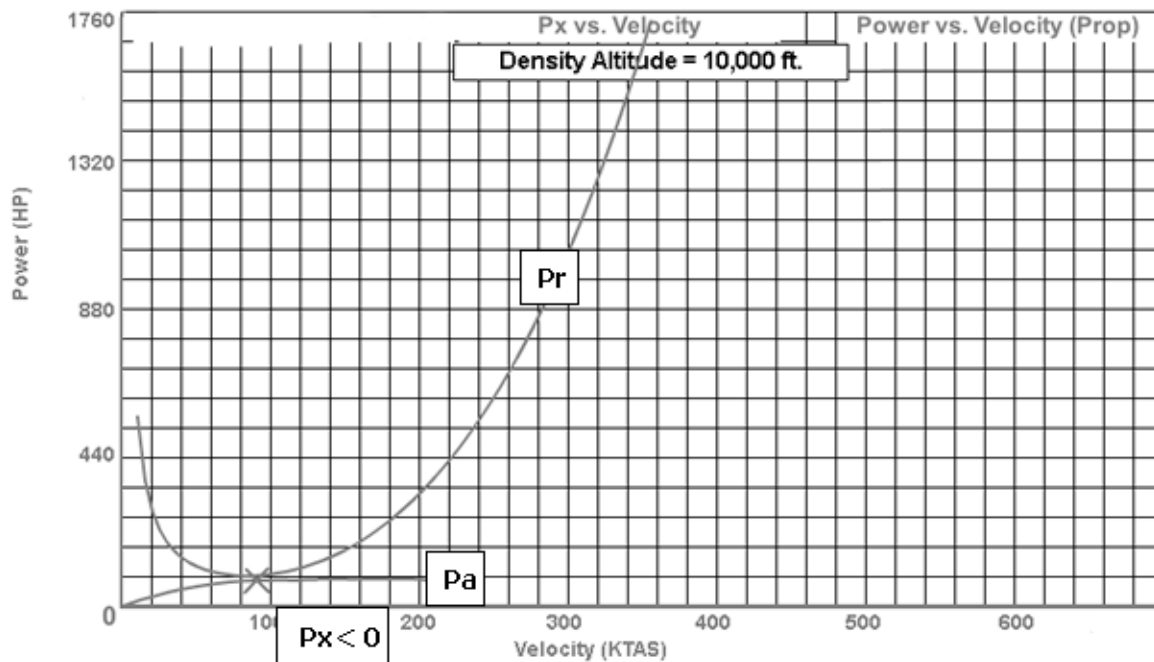


Figure 130

Figure 130 shows that when  $P_a$  is cut in half  $P_x$  becomes less than zero. In this case if one engine quits at 10,000 feet the aeroplane will have to descend. Note that the two engine ceiling is about 18,000 feet, but the one engine ceiling is about 6000 feet.

The diagrams above are approximately correct for a Travelair. The Travelair has a single engine service ceiling of 4200 feet ISA and an absolute single engine ceiling of about 6000', at gross weight. If above its absolute single engine ceiling when an engine fails it cannot climb, and in fact will lose altitude – no matter what the pilot does. We will explore single engine service ceiling more below.

## Wind-milling Propeller Drag

In the graphs above it was assumed that  $P_r$  is the same whether one or two engines are operating. But that is not necessarily so. If the pilot permits the propeller of a failed engine to windmill it creates a LOT of DRAG. As a result the  $P_r$  curve moves up and the shortage of  $P_x$  becomes even more severe than suggested so far. Figure 131 below shows how  $P_r$  changes with a wind-milling propeller.

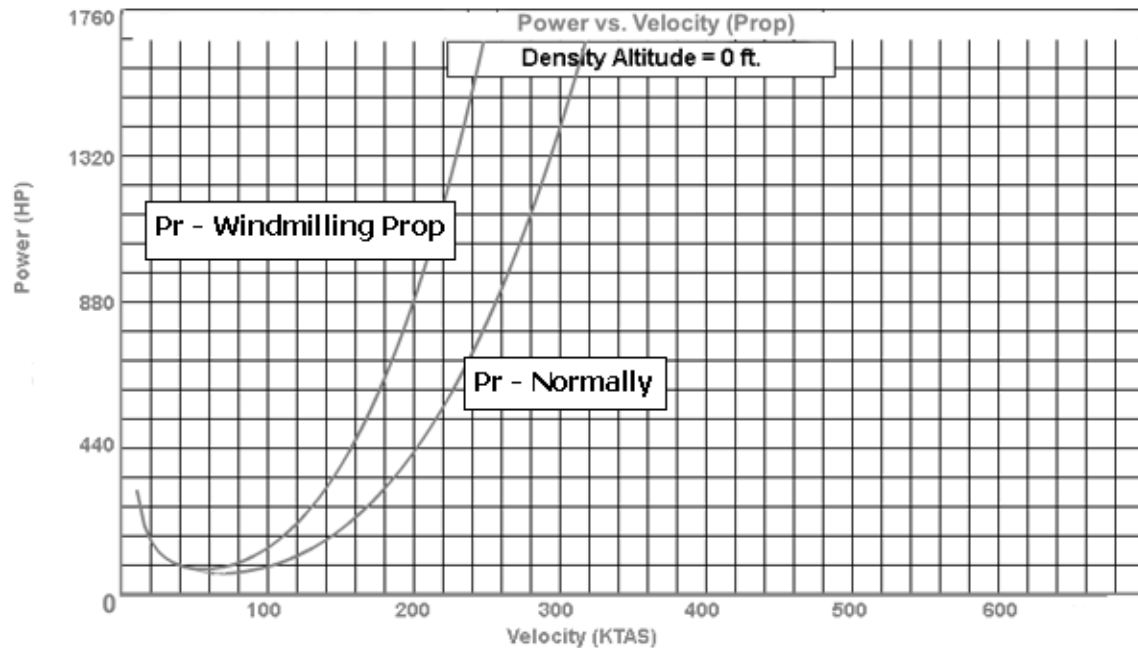
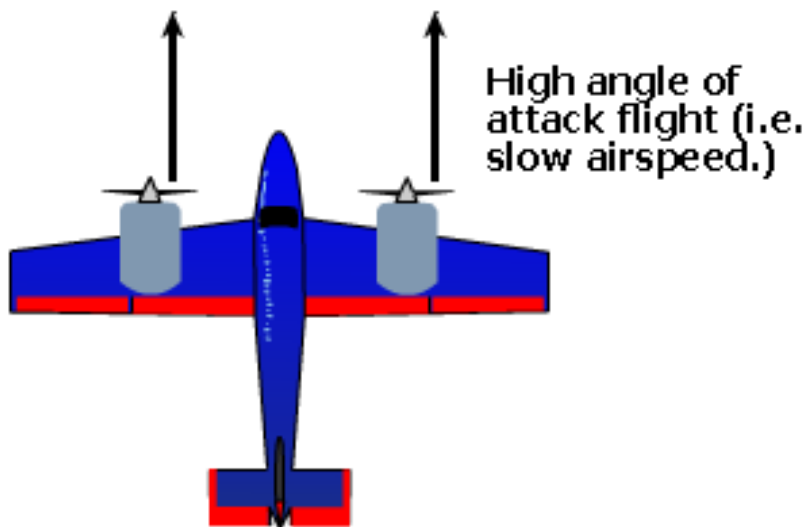


Figure 131

Once the propeller is feathered  $P_r$  returns to the normal value. So, you can see that it is important to feather the engine after an engine failure.

If you think about what we have learned so far you know that there is not much  $P_x$  available after an engine failure. So you can easily use it all up if you allow the propeller to windmill. In the case of a Travelair you should expect the aeroplane to lose altitude, even at sea level. Therefore we **MUST FEATHER** the propeller following an engine failure. If you leave the propeller wind-milling the drag will be so great that it is impossible for you to keep the aeroplane flying.

### ***The Critical Engine Concept***



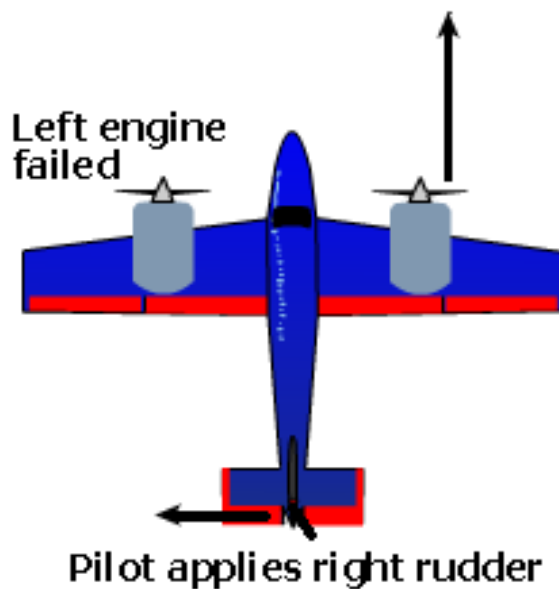
We usually imagine that the thrust produced by an engine manifests itself at the center of the engine. But you learned on page 155 (P-factor) that this is mistaken, especially when flying at high angles of attack. The down-going propeller blade, which is usually the right blade, produces more thrust than the left blade.

The diagram above shows the distribution of thrust for an aeroplane flying at high angle of attack, i.e. at low airspeed such as in a climb. You can see that the right engine causes more yaw than the left engine. As a result the pilot will need to hold right rudder in a climb.

It should be clear that if the left engine fails the situation is worse than if the right engine fails. Therefore, we call the left engine the **critical engine**.

Some aeroplanes have counter-rotating propellers. Normally the left engine rotates clockwise, as viewed from the rear and the right engine rotates counter clockwise. In such a case you can anticipate that you WON'T need any rudder when you climb. You will, of course, need rudder if an engine fails, but there is NO critical engine. (NOTE: there is no critical engine with a jet. NOTE 2: Turbo-props such as King Airs and Dash 8s are almost never counter rotated – so they do have critical engines.)

### Drag Due to Sideslip



Consider the diagram to the left. The critical (left) engine has failed. The aeroplane will yaw to the left and rapidly go out of control if the pilot does not act.

The pilot controls the situation by applying right rudder. The pilot applies just enough rudder to keep the aeroplane straight. The total yaw moment must be zero – in other words the moment created by the engine is exactly offset by the moment created by the tail.

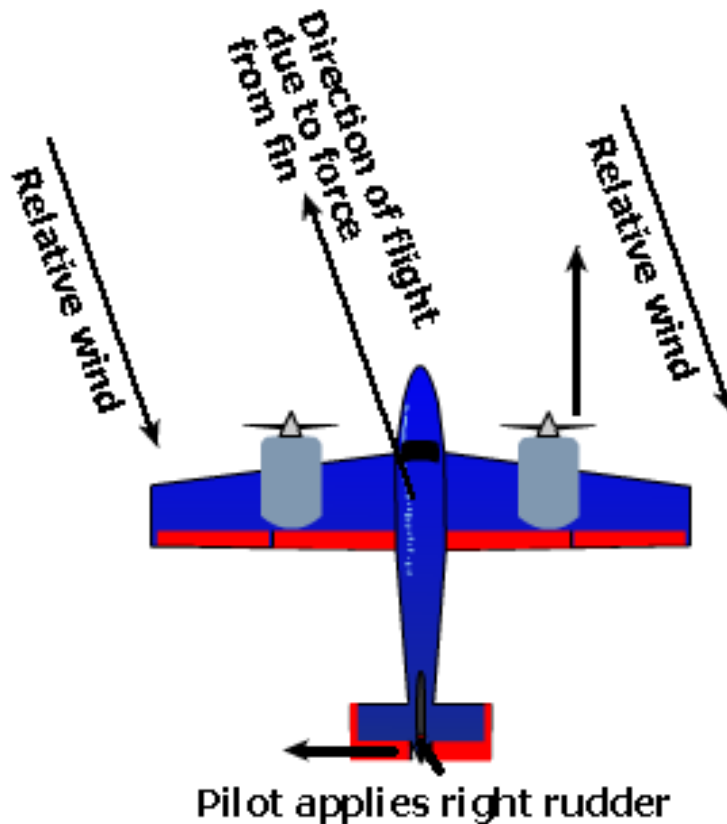


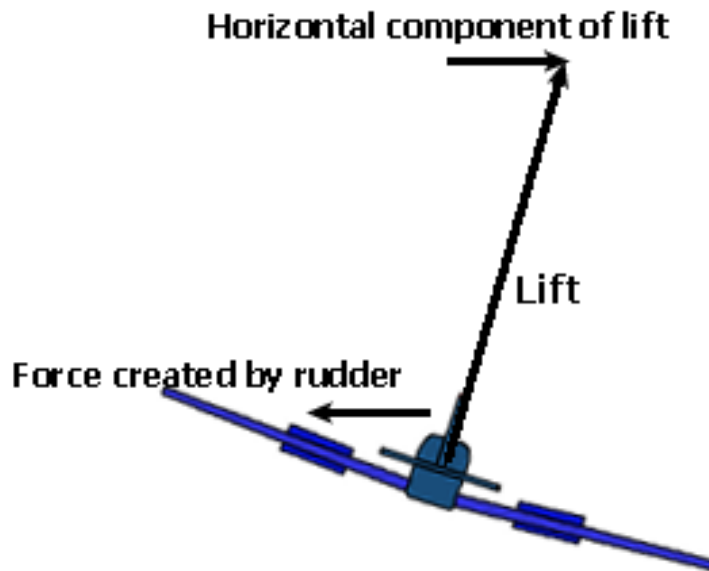
Figure 132

Even though there is no yaw moment there is an unopposed force pushing the aeroplane “sideways.” The force generated by the fin is just like a little rocket motor shooting out the side of the aeroplane. It pushes the aeroplane sideways so that the airflow is no longer parallel to the longitudinal axis of the aeroplane. When air flows across the longitudinal axis we call that a **sideslip**. You can see this situation in Figure 132 above.

There are two problems caused by the sideslip. One is that the effectiveness of the rudder is reduced, so the pilot will need to apply more rudder to keep straight. The second is that the streamlined shape of the aeroplane is effectively distorted. This is a fancy way of saying that parasite drag will increase.

Once again we see that our earlier assumption that  $P_r$  does not change after an engine failure is false. If we allow the aeroplane to slip then drag increases, so  $P_r$  increases. Given the severe shortage of  $P_x$  that is a very bad thing.

At first it may seem there is nothing we can do about this situation. After all we have no choice but to hold the rudder on. If we release the rudder the aeroplane will yaw out of control. What we need is a force in the opposite direction of that created by the fin.



The diagram to the left shows that if we bank slightly toward the good engine a component of lift can be used to offset the fin's force and stop the slip. If we get the bank angle just right there will be NO SLIP and therefore LESS DRAG.

It is crucial that the pilot use a small amount of bank, especially when single engine performance is marginal such as at high altitudes or heavy weights.

Several factors determine the needed angle of bank:

- The fin to CG arm
- Engine thrust output
- Fin size
- Airspeed

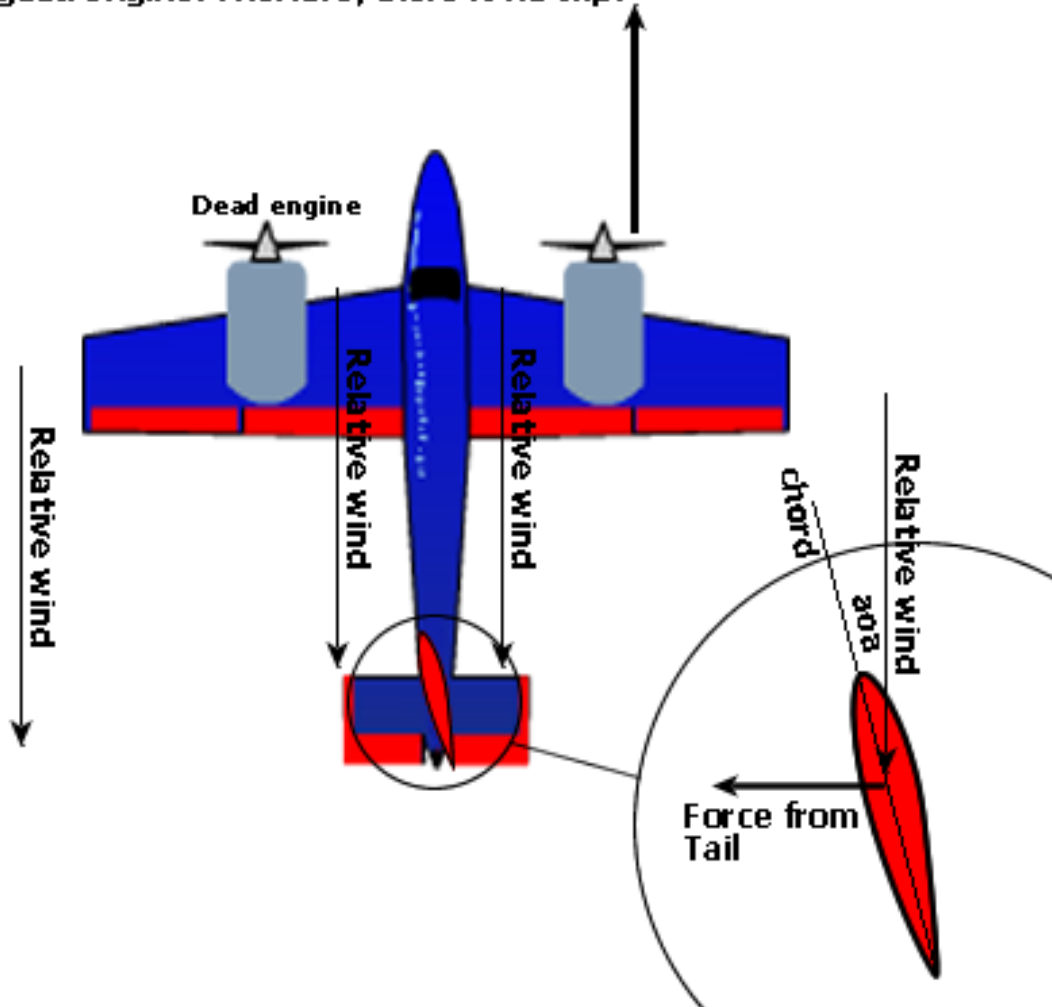
Of these only airspeed and thrust are under pilot control. If the pilot throttles back less bank is needed, but of course the pilot usually doesn't want to reduce power. At high speed less bank is needed. That means that when the pilot slows down to best rate of climb speed bank should be increased.

Rather than attempt to calculate a precise angle of bank the best recommendation is to use 5° of bank. Once the bank is established, if performance is marginal, the pilot may wish to experiment with slightly more or less bank to see if performance improves.

### ***Minimum Control Speed – $V_{mc}$***

As we saw in Figure 132, when flying on one engine the fin must create a force that balances that of the operating engine. Review Figure 85, which shows that for a propeller, thrust is greatest at slow speeds but for a jet the thrust remains constant regardless of speed. For a propeller aeroplane the fin must produce more force at lower speeds and for a jet the fin must produce the same amount of force at all speeds.

**Assume left engine has failed and pilot has banked toward good engine. Therefore, there is no slip.**



**Figure 133**

In Figure 133 the usual fin with a rudder has been replaced by a one-piece vertical tail that rotates when the pilot steps on the rudder. This was done to help you visualize the angle of attack on the combined fin and rudder. This fin is rotated – that is to say the pilot is stepping on the right rudder pedal. The fin is producing just enough force to counteract the failed left engine.

Now consider what the pilot would have to do with the rudder pedal when the aeroplane slows down.

Let's assume the pilot banks toward the good engine the correct amount to eliminate slip. In that case the relative wind is always parallel to the longitudinal axis, as shown in the diagram.

Try to imagine what happens when the aeroplane slows down - just think about what happens with a wing when you slow down; we must keep lift equal to weight, and

to do that we must increase angle of attack. As a consequence there is some speed we cannot go below –and we call that the stall speed.

The same thing happens to the fin. As we slow down we need the same or more “force from tail” (same for jet, and more for a propeller aeroplane.) So, the pilot must increase the angle of attack of the fin. S/he does that by pushing in more and more rudder. Eventually the pilot will have applied full rudder. The aeroplane is now at the limiting speed, which we call  $V_{mc}$ . If the aeroplane goes any slower the fin will not make enough force, so the aeroplane will begin to **YAW out of control**. NEVER fly below  $V_{mc}$  on one engine. (Note – the discussion assumed the fin does not stall before it reaches full travel. This is usually the case.)

There are lots of complications that arise to change the value of  $V_{mc}$  from the one published in the POH. The legal specifications for  $V_{mc}$  are given below. Before looking at them consider the effect of bank angle on  $V_{mc}$ .

In the above analysis we assumed that the pilot maintained the proper bank so that the relative wind strikes the fin along the longitudinal axis of the aeroplane; but what if that is not the case? What if the pilot doesn't bank at all?

In this diagram the left engine has failed but the pilot has NOT banked. The airplane is slipping toward the dead engine.

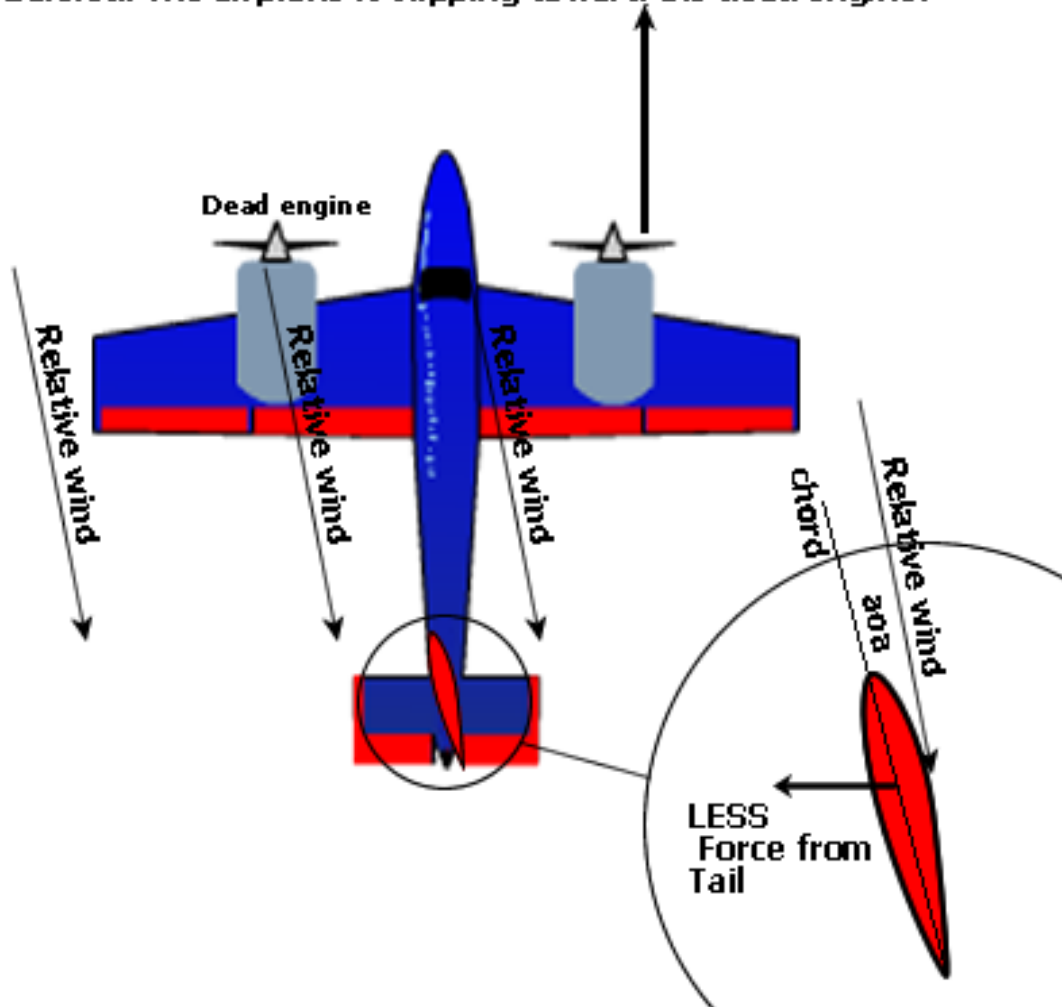
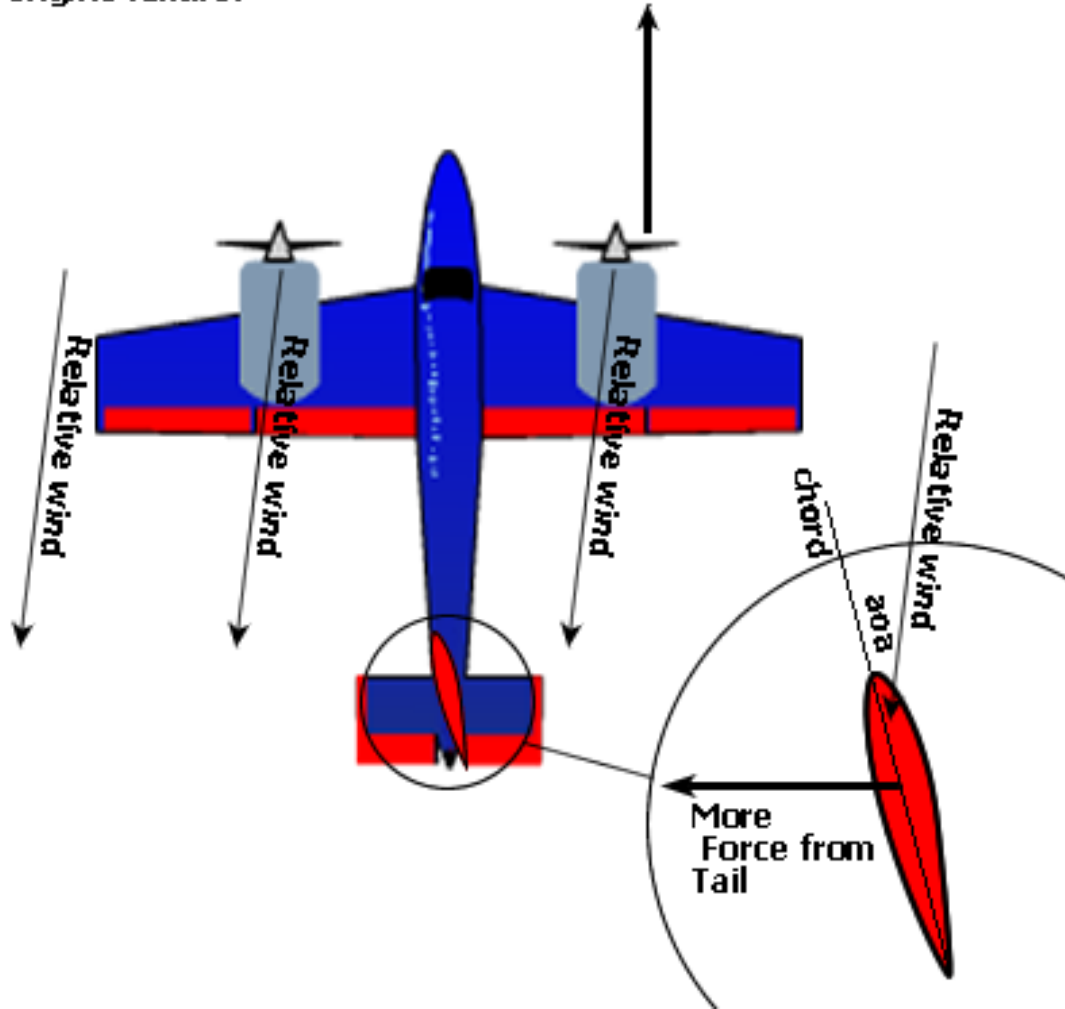


Figure 134

Figure 134 shows the relative wind if the pilot does not bank. The rudder has reached the limit of its travel – i.e. the pilot has his/her foot “on the floor.” You can see that the angle of attack on the fin is reduced and the “force from tail” is less, so we MUST realize that  $V_{mc}$  is **higher**. Tests show that  $V_{mc}$  can be as much as 15 knots higher than the value published in the POH on a powerful turboprop if you don’t bank toward the good engine.



**In this diagram the pilot has banked TOO MUCH following the engine failure.**



**Figure 135**

Now suppose that the pilot banks “too much.” That situation is shown in Figure 135. This time the aeroplane actually slips toward the good engine. This slip will increase drag, so it is not recommended, but it does also increase the angle of attack on the fin. As a result the aeroplane can actually be flown below  $V_{mc}$ . You should NEVER fly below  $V_{mc}$ , but it is very important to note that a substantial bank will help with directional control. On many high performance turboprops there is so much yaw following an engine failure, and the yaw develops so fast, that it is a good idea to bank several degrees, perhaps up to 10, toward the good engine for a few seconds, to help regain control. Once control is established it is ALWAYS wise to reduce bank to the value for no slip (see above), so that drag is minimized.

### **Defining Conditions for $V_{mc}$**

The standards for  $V_{mc}$  are in CAR 525.149.  $V_{mc}$  is based on the following conditions:

## Aerodynamics for Professional Pilots

- Minimum Takeoff weight
- Most unfavorable  $c_g$  (usually aft)
- Critical engine failed – propeller wind-milling unless auto-feather installed
- Operating engine at maximum rated power.
- Gear up
- Flaps set for takeoff
- Maximum five degrees of bank

Note that  $V_{mc}$  is based on maximum power on the operating engine. **The easiest way to regain control when at or below  $V_{mc}$  is simply to reduce power on the operating engine.** If you throttle back you have COMPLETE CONTROL right down to the stall speed.

On most aeroplanes maximum power is only available at sea level. Therefore,  $V_{mc}$  decreases with altitude. At some altitude  $V_{mc}$  becomes lower than the stall speed and ceases to be of concern.

## **High Speed Flight**

We now begin our discussion of high speed flight. In our discussions so far we have assumed the aeroplane was traveling through the air at well below the speed of sound. In that situation air molecules are able to move out of the aeroplanes way and thus air density is more or less unchanged by the passage of the aeroplane. But as velocity approaches the speed of sound the aeroplane will compress the air it is passing through. This substantially changes the way lift and drag develops.

### ***A Brief History of Supersonic Flight***

As World War II progressed aircraft flew faster and faster. By mid-war P-51s, Spitfires and other types were reaching speeds close to that of sound, especially in dives. Pilots began to report control difficulties and unexpected problems, which experts determined were due to flying too close to the speed of sound.

In 1940 NACA commissioned the Bell aircraft company to build a special research aircraft for the purpose of exploring the speed range near and beyond the speed of sound. NACA considered it better to do their research using an aircraft rather than attempt to build a supersonic wind tunnel. They believed technical difficulties in building a supersonic wind tunnel were too complex to overcome. The Germans meanwhile were building a supersonic wind tunnel and doing research in that way.



### **The X-1**

The research aircraft Bell built for NACA was designated X1. Two operational aircraft were finally built, although they did not fly until the war was over. By then the German research data had been captured and several of the questions the X1 was intended to

answer were already known. Nevertheless the X1 became the first aircraft to fly faster than the speed of sound in October, 1947 when Chuck Yeager flew it to Mach 1.1



### **The X-15**

In the 1950s the X-15 became NACA's (later NASA) main high-speed experimental aeroplane. It explored high speed and high altitude regimes to the edge of the atmosphere paving the way for

the space shuttle 25 years later.

### ***Use of Mach Meter***

So far in this text we have discussed speed (velocity) in terms of true airspeed and at times equivalent airspeed. In high speed flight it becomes much more important to know the flight Mach number. We learned how Mach meters work on page 101 (review that material before continuing.)

Pilots of jet aeroplanes normally refer to a conventional airspeed indicator for takeoff and initial climb. As the airplane climbs the TAS increases and the speed of sound ( $a$ ) decreases, due to dropping air temperature. Since Mach number ( $M$ ) is the ratio of  $TAS/a$ . In most aeroplanes the pilots transition from using airspeed to  $M$  as the speed reference in the mid twenty thousands. The exact altitude varies. On modern EFIS equipped aircraft the Mach number is not normally displayed by the computer until it reaches a certain threshold. On older (Type A) Mach meters the Mach scale becomes active and the barber pole begins to move once speed is above the threshold Mach number.

The Mach meter has two important indications. First is the actual Mach number which the pilot references, rather than airspeed, for cruise. The second is an extra needle, commonly called the Barber pole, because it is painted red and white (on an EFIS the display may look somewhat different.) The Barber pole indicates the maximum Mach number for the aeroplane. The pilots must ensure that they never exceed this speed. It is of prime importance when descending since most jets are quite capable of exceeding this speed if the pilots get too aggressive about lowering the nose (diving) in descent.

We will not cover some theory about why the Mach number is so important and then return to our discussion about operating high speed airplanes later.

## ***$C_{Dp}$ Variation in Transonic and Supersonic Flight***

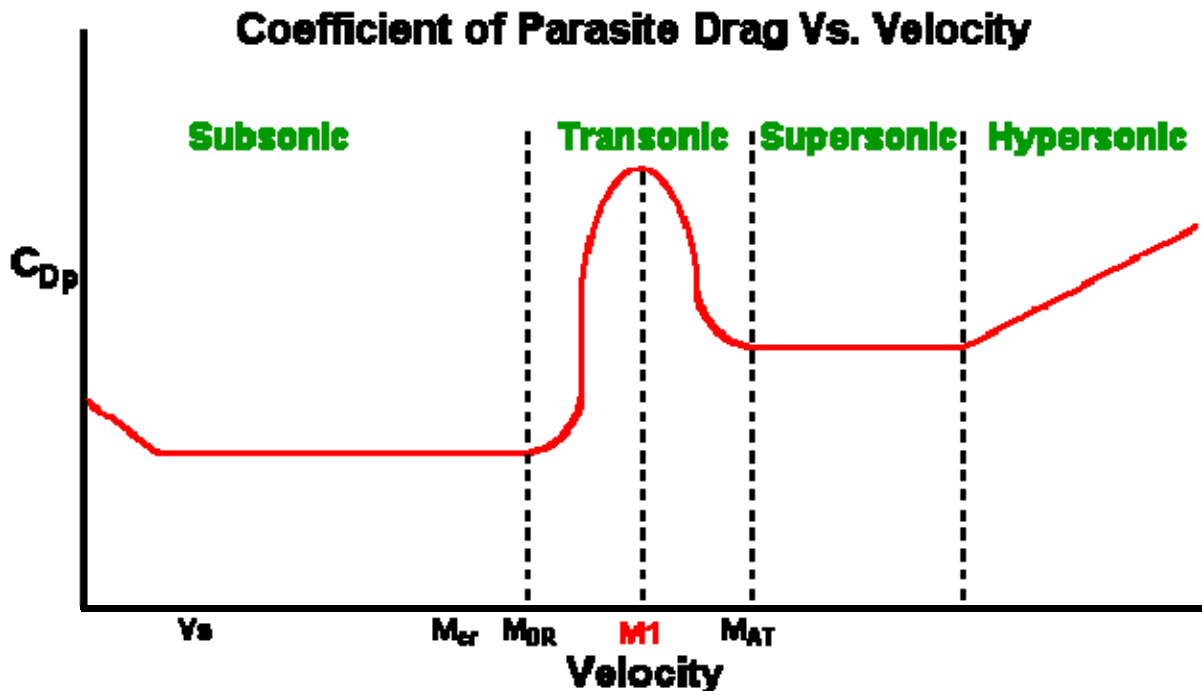


Figure 136

Up to now we have learned that the  $C_{Dp}$  is a constant that depends only on how streamline the aeroplane is. However, when an aircraft approaches the speed of sound  $C_{Dp}$  begins to increase. The Mach number at which  $C_{Dp}$  starts to rise is called the Mach drag rise number ( $M_{DR}$ ) and it always just slightly above the Critical Mach number ( $M_{Cr}$ .)

See Figure 136.  $C_{Dp}$  reaches a maximum at the speed of sound and drops off to become constant again in supersonic flight, but at a higher value than in subsonic flight. When the speed reaches approximately Mach 5 the  $C_{Dp}$  begins to rise again.

The increase in  $C_{Dp}$  above  $M_{Cr}$  takes place because some of the airflow becomes supersonic; this is the beginning of the Transonic-speed range. Transonic flight is by far the worst speed range to fly in. It is much wiser to either remain subsonic, where  $C_{Dp}$  is relatively low, or accelerate fully into supersonic flight, where  $C_{Dp}$  may be higher, but at least lower than in transonic flight.

Before we proceed further we must define some of the terms used above as well as several others.

## **Definitions**

### **Subsonic, Transonic and Supersonic**

We will be using the terms Subsonic, Transonic and Supersonic to describe different speed ranges. In everyday language these terms may have slightly different meanings than the ones we will use. For example the average person uses only two of these terms and defines them as: supersonic means more than the speed of sound and subsonic means less than the speed of sound. Our definitions will be a bit more specific.

The definitions we will use are:

1. Subsonic - All airflow around the aircraft is less than the speed of sound.
2. Transonic - Some airflow is subsonic and some is supersonic.
3. Supersonic - All airflow, around the aircraft, is faster than the speed of sound.
4. Hypersonic – Airflow is faster than Mach 5. At this speed air is heated so much by friction that air molecules ionize. This disrupts radio communications and is the main reason space agencies lose contact with spacecraft for several minutes when they enter the atmosphere on return from space. We will not be considering hypersonic flight in this book.

### **Mach Number**

Mach number (M) is the ratio of TAS and the speed of sound (a.) Therefore, if you are traveling at exactly the speed of sound your Mach number is 1.0. Mach .8 means your speed is 80% of the speed of sound, etc.

The speed of sound changes with air temperature, as shown in Table 1. It is 661.7 knots at 15 degrees Celsius but drops to 573.8 knots at -56 degrees (the typical stratosphere temperature.)

$$M = TAS / a$$

### **Pressure Waves**

As any object moves through the atmosphere it creates pressure waves. These are simply small disturbances caused when air molecules are forced closer together, and then rebound. These disturbances are transmitted through the atmosphere, just like waves passing energy through the ocean. You hear a person's voice for instance, as a result of pressure waves traveling through the air from their mouth to your ear. (Please realize that energy, in the form of waves, moves from their mouth to your ears, but the actual air molecules in the person's mouth do not enter your ear.)

Pressure waves travel at the speed of sound. The speed of sound depends on air temperature, as mentioned above. The blue circles in Figure 137 represent pressure waves

moving out from an aeroplane. This aeroplane is traveling half the speed of sound ( $M = 0.5$ ) and thus it cannot catch up to its own pressure waves.

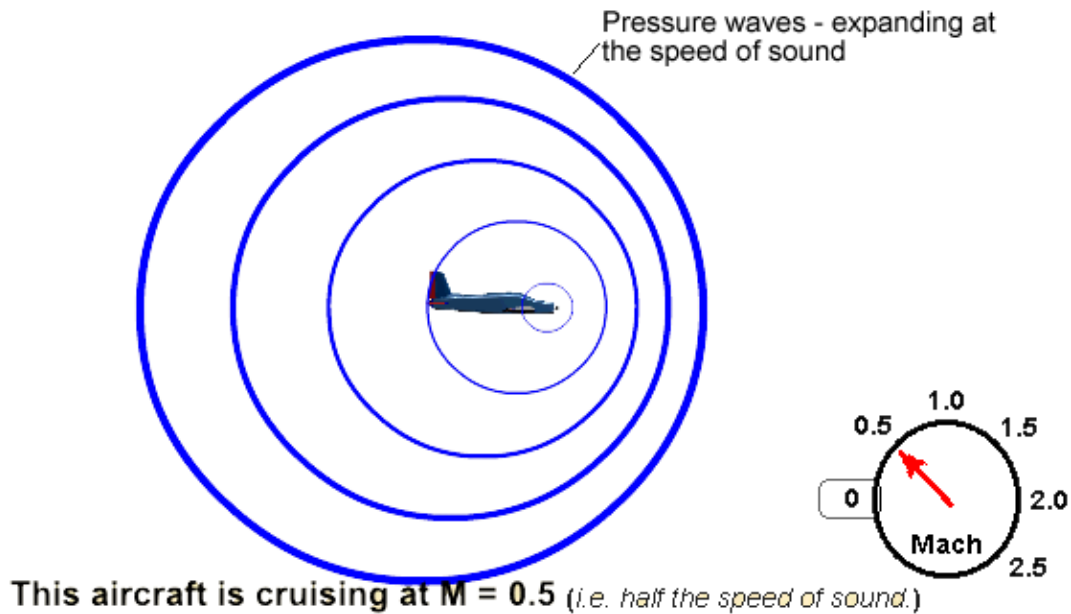


Figure 137

In Figure 137 you see that as long as the aircraft travels slower than the speed of sound the pressure waves move out ahead of the aircraft. One of the effects of these pressure waves is the induced drag that we discussed previously.

### Normal Shockwave

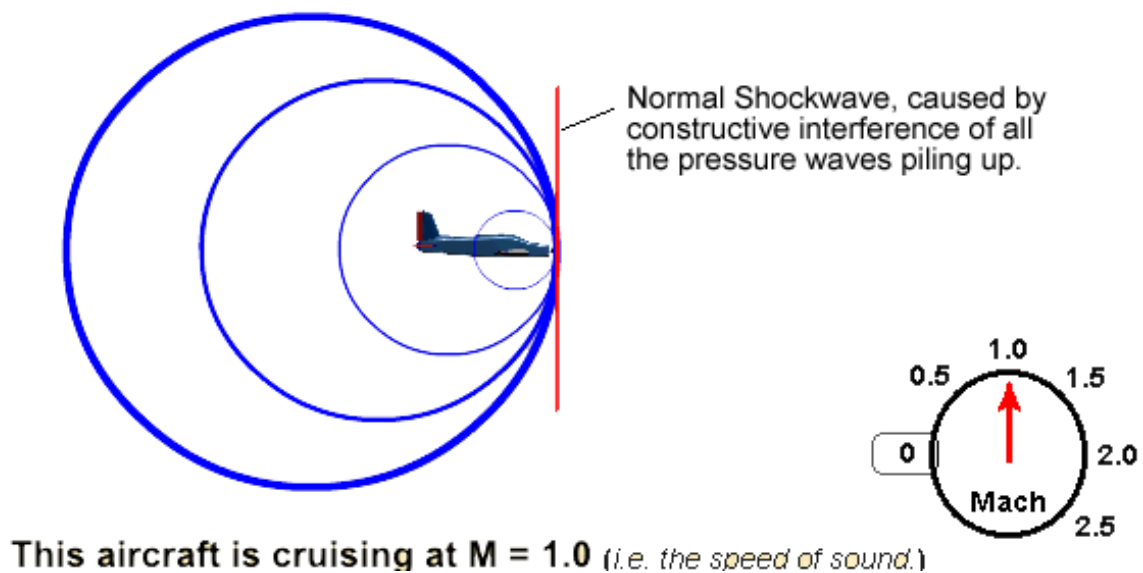


Figure 138



The aeroplane in Figure 138 has accelerated to Mach 1.0. At this speed it just keeps pace with the pressure waves it creates.

Pressure waves continuously emanate from every object moving through the atmosphere. Each one is very small, however when the aeroplane is traveling at the speed of sound or faster they “pile up” along a line, called a **Mach-line** where a process called constructive interference causes their energies to add up. The millions of small pressure waves add up creating a very strong pressure wave, with highly compressed (dense) air, called a shockwave.

At Mach 1.0 the shockwave forms at exactly  $90^\circ$  (Normal) to the airflow. This is known as a **Normal shockwave**.

There is more information about the formation of the Normal shockwave following the definition of Critical Mach number below.

### Oblique Shockwave

Figure 139 shows that if the aeroplane accelerates beyond Mach 1.0 the pressure waves “pile up” along a Mach line that is oblique to the airflow. The resulting shockwave is therefore called an **Oblique Shockwave**.

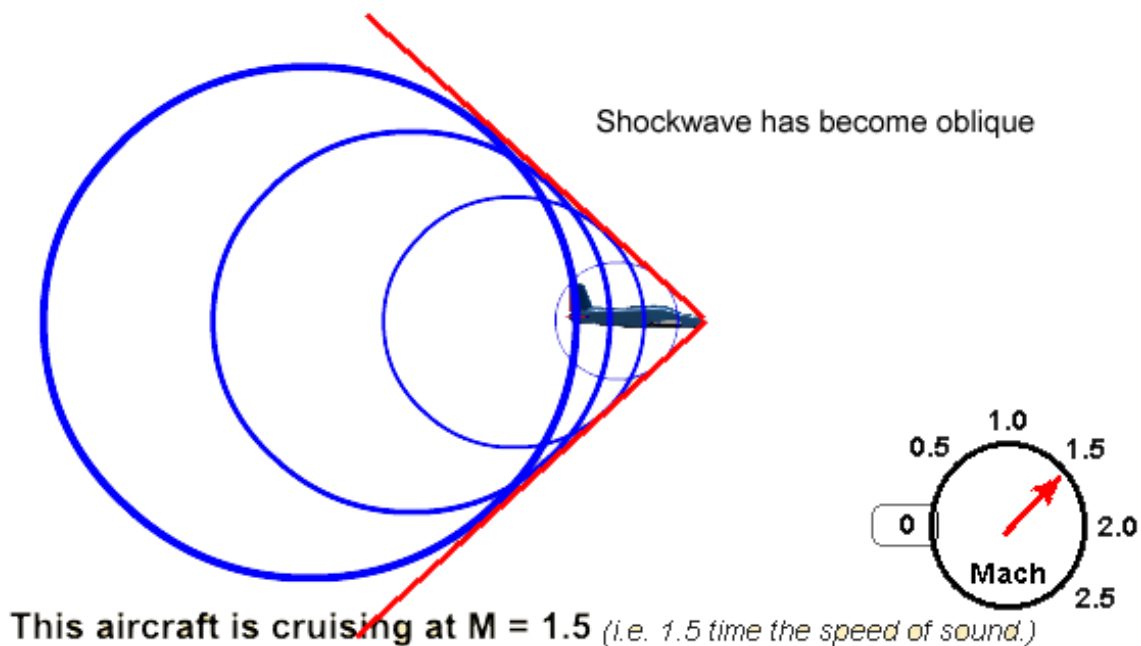
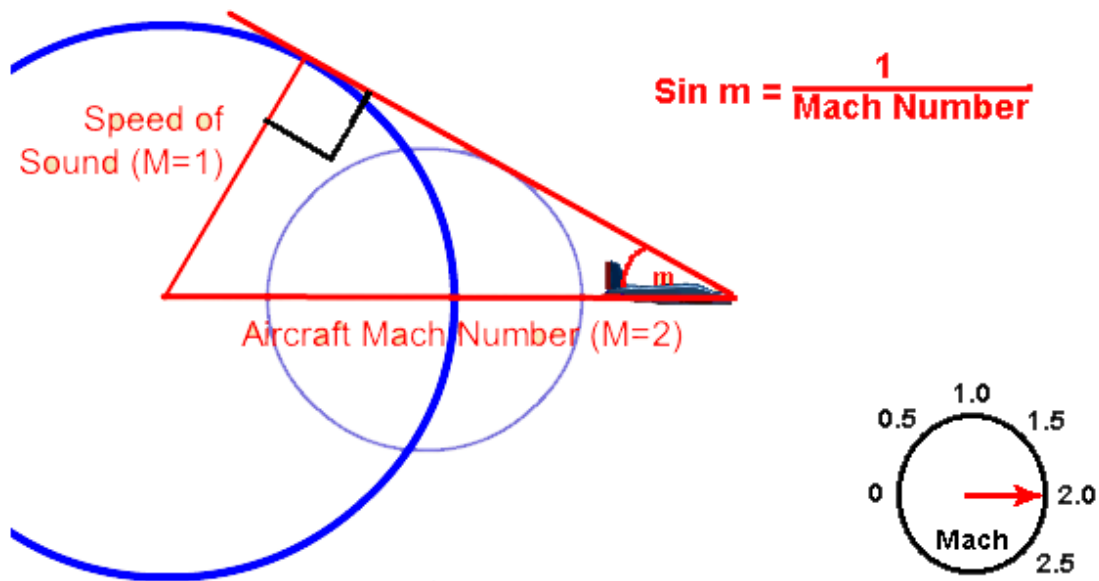


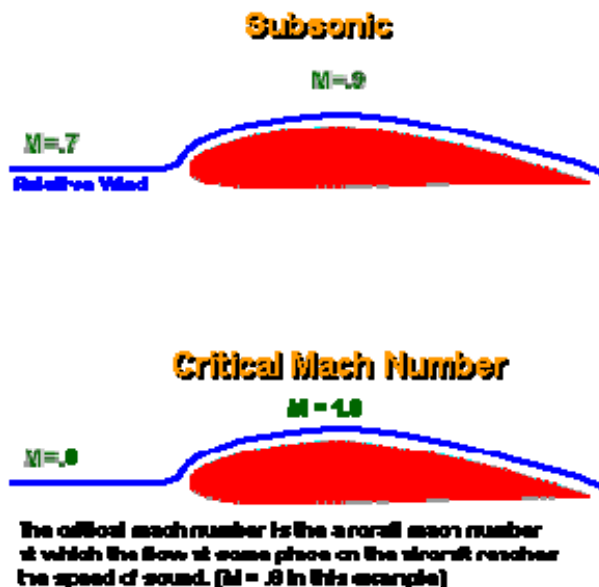
Figure 139

### Mach Angle

We can easily predict the angle of an Oblique shockwave. This is called the **Mach angle**.



## Critical Mach Number

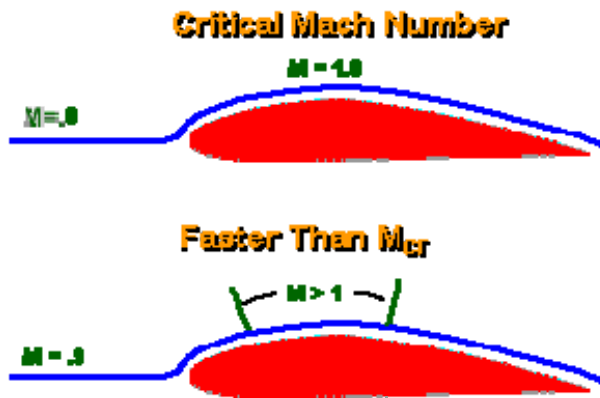


As air passes over a wing it accelerates. We previously learned that this is how a wing produces lift.

If the TAS is close to the speed of sound the airflow may be accelerated to the speed of sound or beyond as it flows over a wing. This effect is shown in the Figure 140. The lower wing is at Mach 0.8 but the airflow has reached Mach 1.0 at one point on the upper surface of the wing.

The Mach number at which the airflow first reaches the speed of sound is called the **Critical Mach number** ( $M_{cr.}$ )

Figure 140



In the lower part of Figure 141 the aircraft has accelerated to Mach .9. There is now an entire region of supersonic airflow over the wing. We say that the aircraft is in Transonic Flight.

Once a significant portion of the airflow is supersonic a "Normal" shock wave will form. Normal shockwaves are the main source of increased drag during Transonic flight and therefore deserve our further attention.

Figure 141

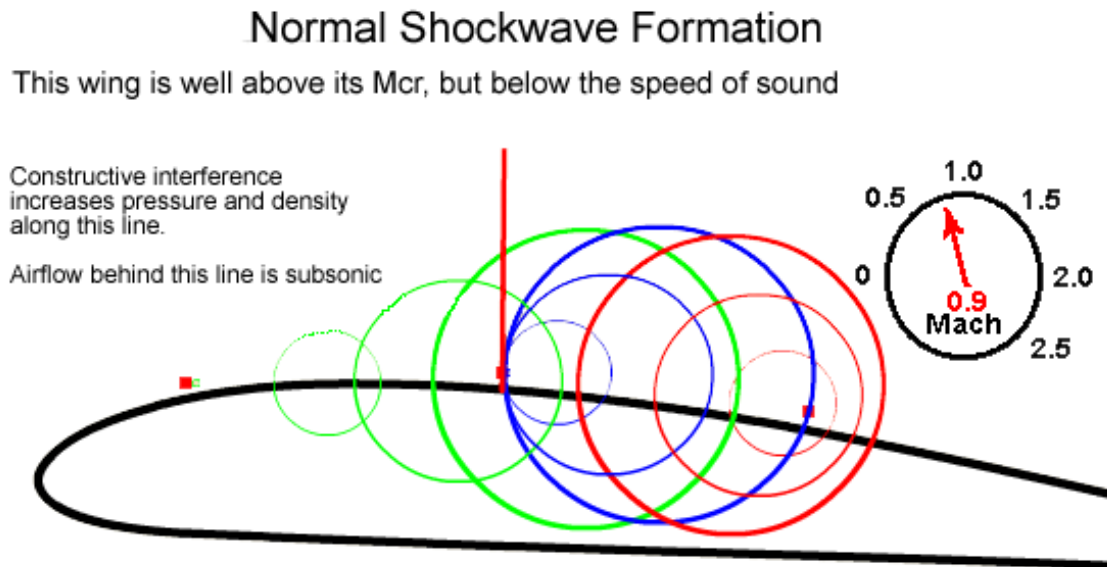


Figure 142

## Formation of Normal Shock Wave

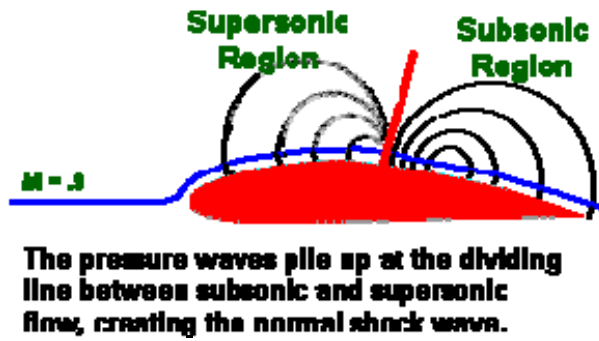


Figure 143

1. A normal shock wave always forms at the boundary between supersonic and subsonic flow
2. The flow behind a normal shock wave is always subsonic

## Transonic Flight

Transonic flight is dominated by the effects of the Normal shockwave.

Normal shockwaves cause drag in two ways:

1. Their very existence requires energy, which is taken ultimately from the engines. This drag is called "Shockwave Drag."
2. Normal shockwave disrupt the boundary layer causing flow separation and thereby effects almost the same as a stall. Thus, the term "Mach Stall" has been coined.

### Mach Stall

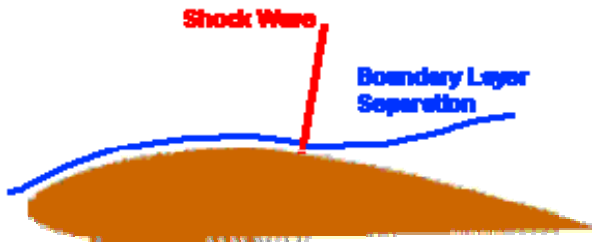


Figure 144 shows a wing with the lift being disrupted and the drag greatly increased by a Mach Stall. In a Mach stall the aircraft experiences a loss of lift, and **increase in drag**, and a tendency to pitch nose down. In other words it has most of the same symptoms as a low speed stall except the angle of attack is quite small and the TAS is very high.

Figure 144

The drag rise in transonic flight is due to flow separation, which is a type of pressure drag. It can be minimized by strategically placing vortex generators at the point where the Normal shockwave first forms. These vortex generators will delay or prevent flow separation, but will not work if the aircraft continues to accelerate well beyond  $M_{cr}$ .

As the aircraft accelerates toward the speed of sound the region of supersonic flow gets greater and greater. Eventually a supersonic flow region forms on the bottom of the airfoil as well. A second Normal shockwave then forms on the bottom of the wing.

As the aeroplane continues to accelerate the region of supersonic flow gets greater and greater and the Normal shockwaves move back toward the trailing edge of the wing.

Once the wing reaches Mach 1 *all* the airflow is supersonic and the Normal shockwaves reach the trailing edge, where they can no longer disrupt the boundary layer. Soon thereafter they become oblique as shown in Figure 146.

### ***Increasing the Critical Mach Number***

Modern jet airliners (except Concorde) are subsonic machines, and therefore must be designed to have high critical Mach numbers. This allows them to remain in subsonic flight, where drag is relatively low, while still flying about 80% of the speed of sound.

We will now look at the wing design features that increase  $M_{Cr}$ .

#### **Low Speed Airfoil**

is quite thick and has relatively large amount of camber. This airfoil produces a low stall speed and low drag up to about Mach = .8



#### **High Speed Airfoil**

is quite thin and has very little camber. This airfoil doesn't produce a lot of lift at low speed but it produces enough lift at high speed with a High Critical Mach number.

This airfoil will probably require ozolic flaps etc.



In order to have acceptable takeoff and landing characteristics.

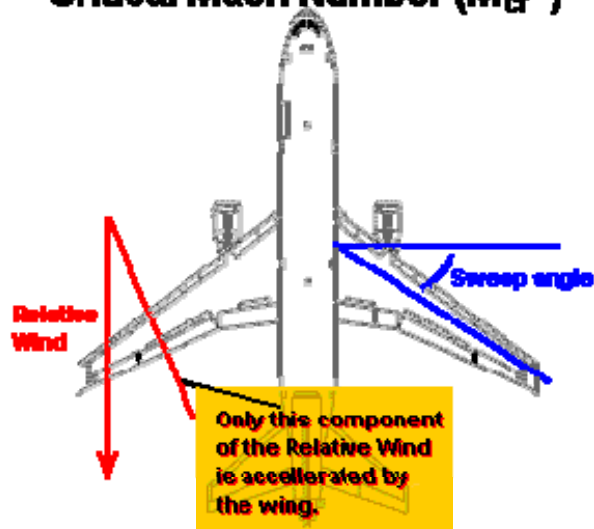
#### **Thin Airfoils with Minimal Camber**

In the 1960's Richard Whitcomb designed the first super-critical airfoil. This airfoil was designed to have a very high critical Mach number (hence the name) and to produce a weak shockwave.

To achieve a high critical Mach number an airfoil must be as **thin** as possible and should have **minimum camber**. It is also best if the location of maximum thickness is as far along the chord as possible.

Before super-critical airfoils were invented the first jet airliners used thin versions of laminar flow airfoils with minimal camber. These airfoils generally meet the criteria stated above. Today airfoils are computer designed to match the exact cruise  $C_L$ , altitude, etc. In all cases however you can expect the design to have the characteristics of thinness, low camber, and carefully tapered thickness distribution.

## Swept Wings Increase The Critical Mach Number ( $M_{cr}$ )



### Swept wings.

When wings are swept back the airflow is accelerated less as it flows over them. Only the component of the airflow perpendicular to the wing is actually accelerated. Therefore, a swept wing has a proportionally higher Critical Mach number.

This is the reason why virtually all modern jets have swept wings.

## Design Features for Transonic Flight

Modern airliners sometimes do exceed their critical Mach number. This is especially true in turbulence or when maneuvering.

### Smooth Air - Normal $M_{cr}$

$TAS < M_{cr}$



### Gust Increases AOA and Causes a Shock Wave to Form



The increased angle of attack (due to the gust) accelerates the air over the wing. This increases the lift but also may push the airflow up to supersonic speed, thereby forming a shock wave.

When angle of attack increases, as in a turn or turbulence, the airflow is accelerated more over the wing. Therefore the Critical Mach number decreases.

An aircraft cruising just below its critical Mach number may experience transient Normal shockwaves in turbulence or if the pilot changes angle of attack with the elevators. This is shown in the diagram to the left.

The main wing is not the only place a shockwave can form, they can also form on the fin or stabilizer, especially when rudder or elevators are deflected just below  $M_{Cr}$ .

Pilots should avoid large control deflections during high speed flight in aircraft not designed for supersonic flight.

Aircraft designers must allow sufficient control margins to ensure that adequate control authority exists so that the pilot can return the aircraft to subsonic flight.

When a Normal shockwave forms potential symptoms are very similar to a stall. The resulting nose down pitch has been dubbed the "Mach Tuck" or sometimes the "Mach stall." (refer to Figure 144.)

If the shockwave disrupts the airflow over the ailerons or the elevators control may be lost. The aircraft will likely pitch nose down and thus accelerate making the situation worse very quickly. Therefore, jet pilots must be ready to throttle back and pull up immediately if an accidental over-speed occurs.

The design features described below are built into most modern jets to provide the extra control margin necessary to get the aeroplane back under control and slowed down to below its Critical Mach number.



### Trimming Tail

It is crucial for a jet airliner that might encounter transonic effects to have a stabilizer and elevator combination that will work effectively. Preventing a Normal shockwave is imperative.

A Normal shockwave is most likely to form at the hinge-line between the elevator and stabilizer if the pilot deflects the elevator when near  $M_{Cr}$ .

Conventional stabilizers have a non-moving horizontal stabilizer and a trimmable elevator. This promotes the formation of shockwaves because they become highly cambered when the elevator is deflected. Once a Normal shockwave forms it disrupts the airflow over the elevator rendering it useless, or at least greatly degraded in effectiveness.

Therefore, almost all jets use either a fully moving tail surface, known as a Stabilator or a trimmable stabilizer, such as the one shown in the photo above.

A trimmable stabilizer is controlled by the pilot through the trim wheel in the cockpit. When the pilot trims the aeroplane the elevator and stabilizer are brought into alignment so that there is no camber on the tail surface (i.e. the elevator and stabilizer remain in alignment regardless of the trimmed angle of attack.) This is ideal for preventing shockwaves.

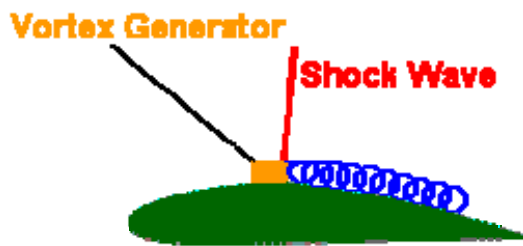




## Vortex Generators

Vortex generators are small airfoils attached to the surface of a wing, fuselage, fin, etc. They catch the free stream airflow (i.e. above the boundary layer.) These vortex generators do exactly what their name implies, they generate a vortex.

The vortex re-energizes the boundary layer thus preventing flow separation. Without them there would be loss of lift and increased drag.



**The vortex from the Vortex Generator re-energizes the boundary layer thus preventing flow separation.**

**Flow separation would cause a loss of lift and an increase in drag (Parasite Drag.)**

Vortex generators are very effective in transonic flight since there is a lot of energy in the free stream (it is supersonic after all.) To be effective the vortex generator must be placed at the point where the Normal shockwave forms. The energy added to the boundary layer then gets the boundary layer through the shockwave and thus reduces the negative effects of the shockwave. The location of the Normal shockwave shifts back if the aeroplane continues to accelerate, therefore vortex generators are only effective when the shockwave first forms and provide a momentary protection to allow the pilot to slow the aeroplane, they do not provide ongoing protection if the aeroplane continues to accelerate.

Most modern jets have some vortex generators; sometimes on the wing, sometimes on the fin ( ahead of the rudder) and sometimes on the fuselage (if it is found to be form shock waves.)

## Powered Controls

Large aeroplanes usually need power assisted controls even if they don't reach the speed of sound. Many business jets are small enough to fly with only cable and pulley (or pushrod) controls even at fairly high speed. Never-the-less and aeroplane that may encounter shockwaves in flight usually requires some form of power assisted controls to provide the pilot with enough force to deflect the controls in the presence of shockwaves.

Power controls will always be needed for supersonic flight, as we will discuss later.

### **Transonic Design Feature Summary:**

Modern jet transports have swept wings to promote high  $M_{cr}$ , thereby increasing the usable subsonic speed range. Other design features, such as vortex generators and trimmable stabilizers, are intended to protect the aircraft from loss of control due to inadvertent incursions into transonic flow. Power assisted controls are also almost always provided to assist the pilot to move the elevators, ailerons and rudder against the large forces that develop at high speed.

Below is a list of design features common on transonic aeroplanes:

1. Swept wings to increase  $M_{cr}$
2. Thin wings to increase  $M_{cr}$ .)
3. Minimum airfoil camber to increase  $M_{cr}$
4. Vortex generators to prevent drag and control loss due to airflow disruption.
5. Trimmable tail (or a Stabilator) to minimize risk of shockwave formation on the tail.
6. Boosted controls, or fly by wire, to provide control power needed to overcome high forces.

### ***The Coffin Corner***

The coffin corner is a feature of high speed, subsonic flight that is unique to the high flying modern jet aircraft. Pilots of slower low flying aeroplanes never experience this phenomenon.

Every aeroplane has a stall speed, and as we learned on page 46 the true stall speed increases with altitude. For a typical jet transport aeroplane with a stall speed of 160 KEAS the true stall speed at 45,000 feet is 364 KTAS. The same aeroplane will have a critical mach number of perhaps 0.9, which is 516 KTAS. Consequently the aeroplane MUST be flown between these two speeds. If it flies slower than 364 KTAS it will stall, but if it flies faster than 516 KTAS it will experience shockwaves that may cause a “Mach stall.”

Now imagine that the pilot is forced to maneuver rapidly (perhaps to avoid traffic) or the aeroplane experiences turbulence that exerts a g-force aeroplane. The result will be an increase in the stall speed and a reduction in the critical mach number. In a 45° bank turn the stall speed would increase to 433, which is Mach 0.75. If the critical Mach number declines to 0.85 the “wiggle room” for the pilots is getting quite small (less than 60 knots from stall to mach limit.) This phenomenon has been dubbed the coffin corner. The corner grows narrower and narrower the higher you fly. The concept is shown in Figure 145.

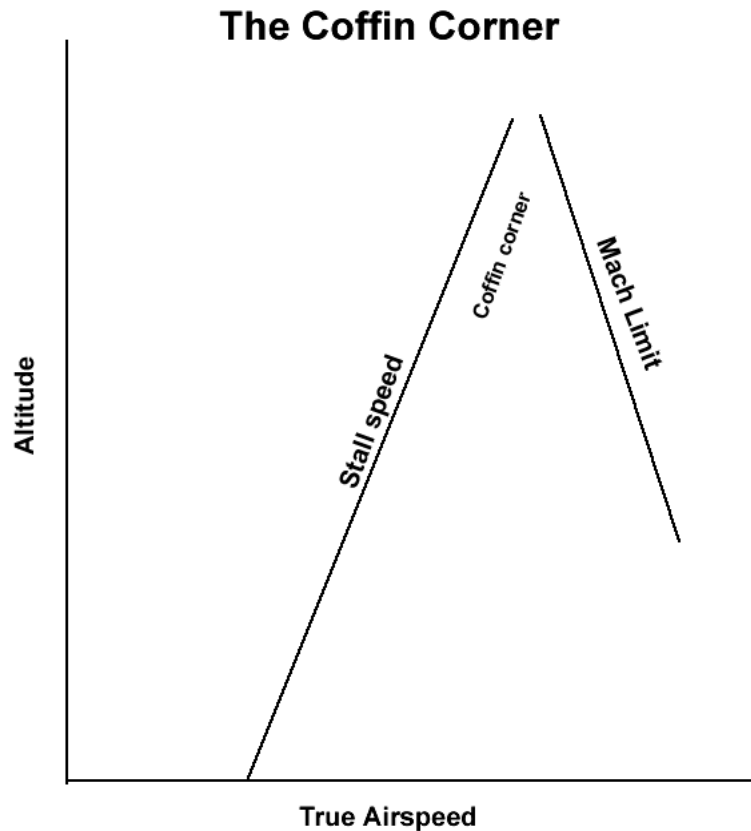


Figure 145

The coffin corner is a phenomenon unique to high speed subsonic aeroplanes. In the future transport aeroplanes may be designed to exceed the speed of sound (as Concorde used to) but until then the coffin corner is a real concern for jet pilots. To maintain an adequate safety margin flight at high altitude is limited to gentle turns only. Pilots normally do not exceed  $25^\circ$  of bank when turning. To maintain a reasonable margin some aeroplane may have to fly at a slightly lower altitude than they would without this concern.

The only way for an aircraft designer to increase the operating altitude of a jet aeroplane is to increase the critical Mach number or reduce the stall speed. Since it is the stall speed in cruise configuration that matters the only practical way to achieve reduced stall speed is to limit wing loading. Some business jets, which almost always have lower wing loadings than transport jets, are certified to fly above 50,000 feet. It is difficult to achieve the same altitudes in a transport jet due to the coffin corner effect.

Next we will examine supersonic flight. Supersonic flight is not yet a viable commercial proposition, but it may be in the future.

## Supersonic Flight

Supersonic Flight is dominated by oblique shockwaves.

The drag caused by Normal shockwaves, which dominated transonic flight, disappears when supersonic. Therefore, it is not surprising that drag actually decreases in Supersonic flight.

Once an aircraft is above  $M=1.0$  shockwaves become oblique, but instead of one pair there are two (see Figure 146.)

Once the aeroplane exceeds Mach 1.0 a bow shockwave forms ahead of the wing. This is due to the air particles "smashing" into the leading edge of the wing. In subsonic flow this does not happen because the pressure wave "steer" the air allowing it to flow smoothly around the leading edge. But, in supersonic flow that cannot happen. The resulting bow wave is like the wake seen coming from the prow of a ship in the water.

The bow wave creates an area of high pressure at the leading edge of the wing (called the stagnation point, meaning point where air slows.) Remember that we previously saw that pressure drag in subsonic flight is due to low pressure behind the aeroplane, but here we see that a lot of drag is due to high pressure ahead of the aeroplane in supersonic flight (which when you think about it is more intuitive than the subsonic situation.) The high pressure at the stagnation point needs to be minimized. The bow-wave and the trailing oblique shockwaves are shown in Figure 146.

## Oblique Shockwaves

Just above Mach 1 a Bow-wave forms. As the wing accelerates to mach 2 both the bow wave and teh trailing edge waves become oblique. (At Mach 2 the mach angle is 30 degrees)

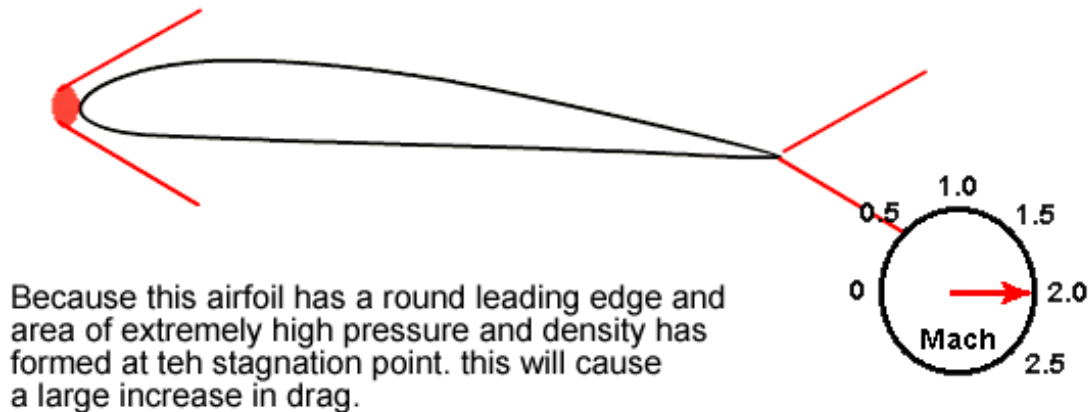


Figure 146

The Normal shockwave that forms in transonic flight moves back to the trailing edge of the wing when the aircraft reaches the speed of sound. In that position it posses relatively little problem. Once the aircraft accelerates beyond the speed of sound the trailing shockwaves remain attached to the wing and sweep back becoming a weak oblique shockwave. This trailing oblique shockwave takes energy to produce, and thus causes some wave drag, but it cannot affect the boundary layer and therefore has no effect on pressure drag.

*All Oblique shockwaves have the characteristic of slowing the airflow passing through them, but **not** to subsonic speed. The airflow behind a Normal shock wave is always subsonic, but the airflow behind an Oblique Shock Wave is still **supersonic**.*

The primary drag problem in supersonic flight comes from the **bow wave** ahead of the wing. The bow wave causes an area of very high pressure just in front of the wing. This causes a large increase in pressure drag.

### Minimizing the Effect of Bow Waves

There are two ways an aircraft designer could deal with Bow waves:

1. Use a "Supersonic Airfoil"
2. Sweep the Wings

### Ideal Supersonic Airfoils



### Supersonic Airfoils

One solution to the drag caused by bow waves is to make the leading edge of the wing very sharp, like the prow of a boat. This design feature allows the bow wave to attach to the leading edge thus eliminating the area of high pressure ahead of the wing.

The problem with sharp leading edges is that they are very poor in subsonic flight because the aircraft will have very high stall speed which necessitates very long runways. Consequently these airfoils are unsuited for use on airliners. However they are OK for a missile.

In summary – sharp leading edge airfoils are widely used on missiles and other supersonic devices where takeoff and landing are not required. But for an aeroplane slow speed performance cannot be ignored so supersonic airfoils is not a practical solution.

Figure 147



Figure 148 shows how a supersonic airfoil generates lift. An oblique shockwave forms at the leading edge, which deflects the air flow. If the wing has a positive angle of attack the flow is slowed along the bottom of the wing, which increases pressure there. Thus lift on the front half of the wing is due to increased pressure below the wing.

At the midpoint of the wing the air flow must negotiate a bend. The air flow on the upper side of the wing must accelerate. An interesting thing to note is that while subsonic flow would likely separate (form eddies) at such a sharp corner the high energy of the compressed air surrounding the supersonic airfoil easily makes the turn, accelerating as it does so. Bernoulli's equation still applies, so static pressure decreases when the air accelerates. Consequently lift on the aft half of the wing results from a drop in static pressure above the wing (similar to subsonic flight.)

The equal contribution to lift by the front half of the wing shifts both the aerodynamic center (ac) and the center of pressure (CP) to the midpoint of a supersonic wing, see Figure 149.

## Supersonic Airfoils and Expansion Fans

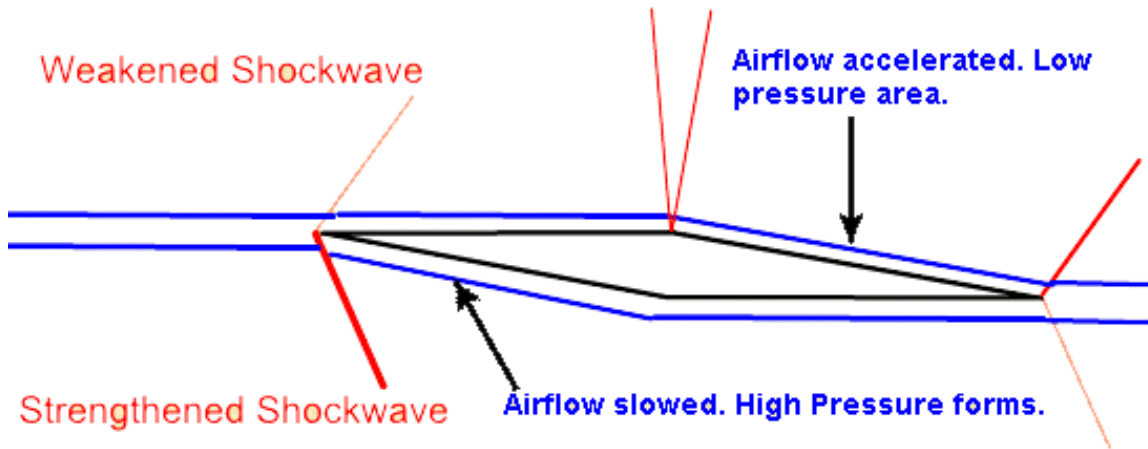
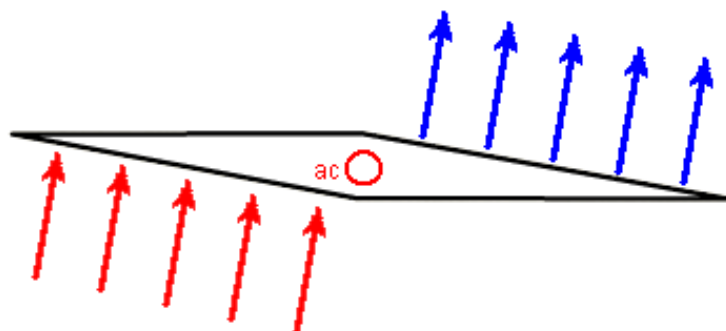


Figure 148

## Supersonic Airfoils and Expansion Fans

In supersonic flight the aerodynamic center moves to the 50% chord point



The wing is supported by high pressure under the front of the wing and low pressure above the back half.

Figure 149

## Swept Wings in Supersonic Flight

As mentioned above, swept wings are the second option an aircraft designer has to minimize drag caused by the bow wave.

Earlier we learned that swept wings raise the critical Mach number thus making modern airliners able to fly at higher subsonic speeds. But sweep also reduces the effect bow waves in supersonic flight.

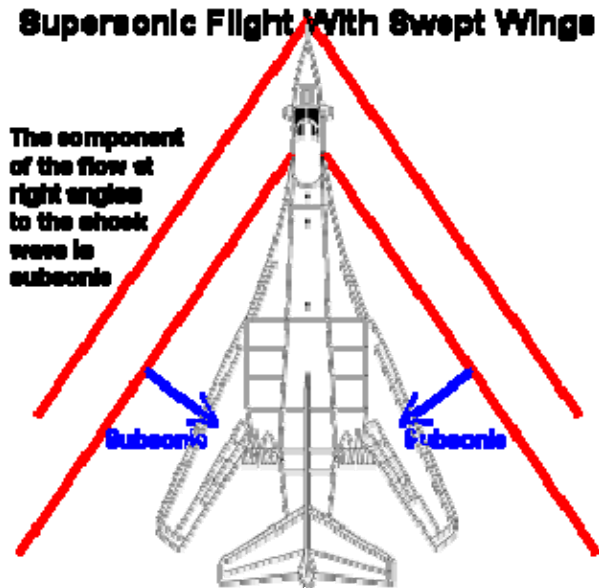


Figure 150

Every part of an aeroplane which strikes the airflow and slows it to subsonic speeds produces a shockwave (a bow wave.) These bow waves sweep back at the Mach angle. The bow waves are of course Oblique shock waves.

Earlier we learned that the airflow behind an Oblique shockwave is supersonic. However, the component of the airflow at right angle to the Oblique shockwave is subsonic. This concept is shown in Figure 150 and Figure 151.

If a wing is placed behind an oblique shock wave and parallel to it, as shown in Figure 150, then the **air flowing over that wing is subsonic**, even though the aircraft is flying faster than the speed of sound. Therefore, a subsonic airfoil, with round leading edges can be used without creating a bow wave.



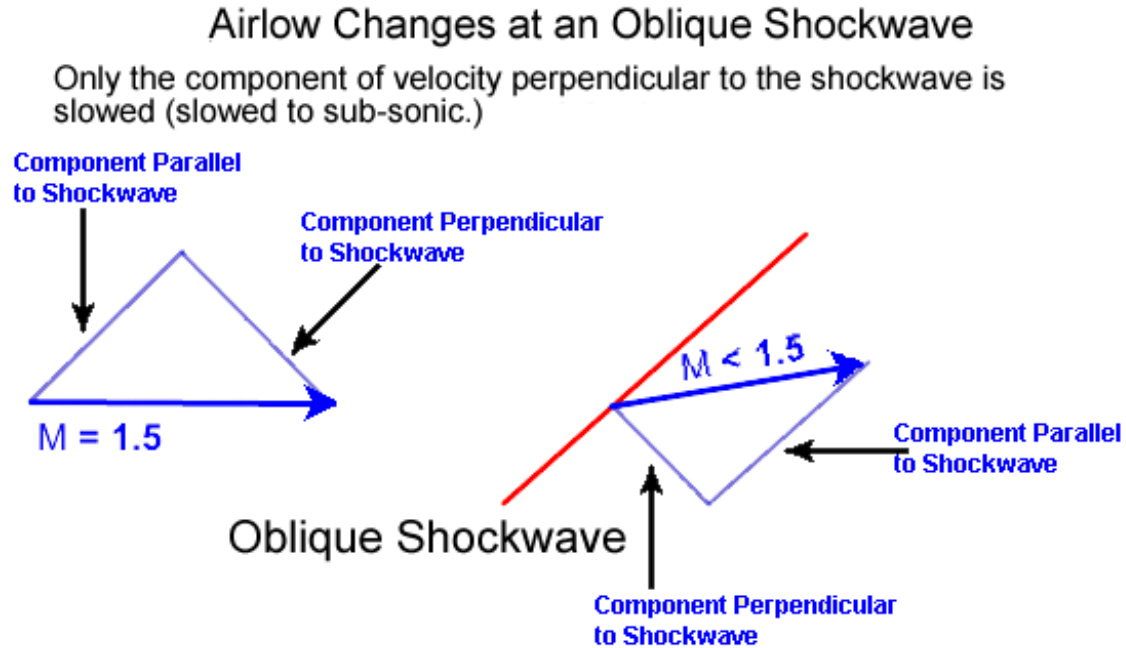


Figure 151

In order for swept wings to be effective in supersonic flight they must sweep back at least to the Mach angle. For example at Mach 2 the wings must be swept at least 30 degrees. As long as the designer can accommodate this requirement the wing will react as though it is in subsonic flight and a conventional airfoil, with a round leading edge, can be used thereby producing reasonably good slow speed flight characteristics. This is the design strategy of choice for most modern supersonic aircraft.

The primary advantage of using swept wings with a round leading edges is that low speed characteristics, including stall, are good and therefore takeoff and landing are good. But at some degree of sweep the takeoff and landing characteristics begin to deteriorate. The problems include:

1. A very large stalling angle of attack which requires extreme nose-up attitude for landing
2. Excessive lateral stability, which contributes to Dutch Roll problems.

Some aeronautical engineers feel that these problems limit the future use of swept wings for supersonic flight.

Of course variable geometry wings – i.e. designs with wings that sweep back for supersonic flight and forward for landing and takeoff are the obvious answer. To date this solution has only been used on a few military aeroplanes, such as the one in Figure 150. The problem with variable geometry is the technical complexity and weight that the

mechanism adds to an aeroplane. It is hard to predict whether this solution will make its way into civilian aeroplane design in the 21<sup>st</sup> century.

## ***Longitudinal Stability in Supersonic Flight***

### **Aerodynamic Center Moves Back In Supersonic Flight**



Review the effect of CG on longitudinal stability, as explained on page 147.

In subsonic flight a wing has an aerodynamic center at close to 25% chord. When an aircraft accelerates into supersonic flight the ac moves back to 50% chord. The reason can be seen in Figure 149, which shows that lift is evenly distributed on a supersonic wing. There are two effects:

### **Mach Tuck**

The nose of an aeroplane always tends to pitch nose down when it transitions from subsonic to supersonic. This tendency is

called "Mach Tuck." Aircraft designed for supersonic flight, such as Concorde, have computerized "Fly by Wire" systems that automatically compensate for this, so the pilot never notices the trim change. But, an aircraft not designed for supersonic flight (like most airliners) pitches down significantly if forced through transonic into supersonic flight. Such aircraft require a Mach-over-speed warning system (known to pilots by the slang expression "hen pecker.") When a pilot hears the over-speed warning the procedure followed is essentially the opposite of a stall recovery – power is reduced to idle and the nose is pulled up firmly.

## **Longitudinal Stability Increases in Supersonic Flight**

We learned previously that longitudinal stability depends upon the center of gravity being ahead of the aerodynamic center. The aerodynamic center moves back in supersonic flight and this increases longitudinal stability. Increased longitudinal stability means the aeroplane resists changes in angle of attack. Pilots will notice that the elevator control becomes heavy and less responsive. This phenomenon was first encountered during the Second World War when some P-51 and Spitfire pilots got too close to the speed of sound, usually in a dive; the pilots experienced a tendency for the nose to pitch down and even more alarmingly it took all their strength to pull the nose back up. Some did not make it, diving into the ground; others aircraft broke up when they exceeded their maximum design speed.

Modern supersonic aircraft have little difficulty passing through the sound barrier because they are designed with very large and power assisted elevators (or Elevons.) A nose up trim change is always required on transition to supersonic flight (the fly-by-wire system makes the trim change automatically without the pilot needing to take any action.) Even though an aeroplane designed for supersonic flight has no difficulty jet pilots must realize that an aeroplane not designed for supersonic flight has little chance of successfully flying much beyond its critical Mach number.

### **Area Rule**

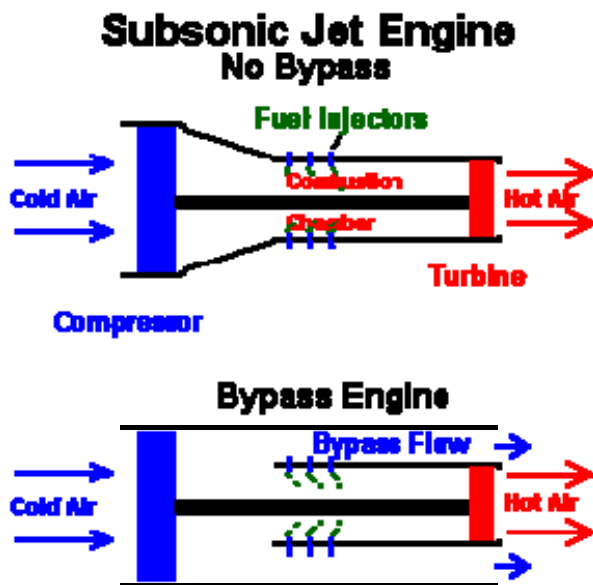
The primary difference between supersonic and subsonic flight is that air is compressed (made more dense) by a passing aeroplane in supersonic flight. The amount of compression is obviously directly related to the volume of the aeroplane.

Imagine the following as seen from the perspective of a parcel of air assaulted by a passing (poorly designed) supersonic aeroplane. As the aeroplane plunges in its sharp nose begins to compress the air, then the wider fuselage compresses the air more, *suddenly* the wing volume rapidly adds to the compression. After the wing passes an immediate reduction in the aeroplane's cross-sectional area causes a rapid decompression, only to be recompressed by the fin, stabilizer, and aft fuselage. Finally the aeroplane is gone and the air rebounds to its original pressure and density.

The above describes what would happen if you took an aeroplane designed for subsonic flight and forced it supersonic. It is a bad design however. A properly designed supersonic aeroplane has cross-sectional areas that smoothly increase to a maximum at mid length and then drop off smoothly to zero at the tail. If done properly the air is smoothly compressed then decompressed once and only once. The procedure is called **area rule**.

It is much easier to achieve area rule on a tailless design like Concorde than on one with separate wing and tail. However any design can be area ruled. The result is a fuselage with a "waist" where the wings and tail join so that the total cross-section area does not change rapidly at any point.

## Engine Design for Supersonic Flight



Conventional jet engines have a compressor section that uses fans to compress the air before it enters the combustion section of the engine.

Supersonic jet engines have the same basic engine design but there are a few extra considerations.

There is a type of engine known as a RAM jet that only works in supersonic flight. They are fine for missiles, but not useful for a passenger carrying jet. We will briefly cover how they work at the end of this section.

### Bypass Engines in Supersonic Flight

Bypass means that some of the air accelerated by the compressor does not go through the combustion chamber, instead it provides thrust directly just like air accelerated by a propeller.

Bypass will not work in supersonic flight for the same reason propellers do not work. The power it would take to accelerate air to supersonic speed with a propeller (or fan) is too great. Therefore, engine developers are working toward an engine on which bypass can be turned on and off for subsonic and supersonic flight.

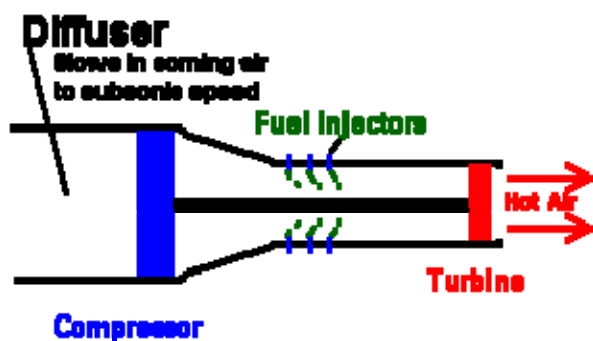


Figure 152

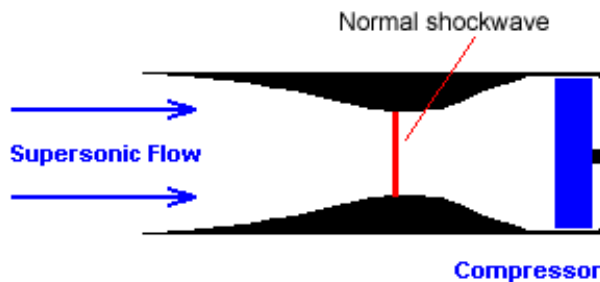
## ***Supersonic Diffusers***

Compressor fans will not work efficiently on supersonic air. Therefore, the airflow must be slowed to subsonic speed before it enters the compressor section of the engine. This is the job of the engine inlet, also known as a diffuser. The diffuser resides ahead of the compressed section as shown in Figure 152

A diffuser can be quite simple or very complex, depending on a number of factors including how broad the desired supersonic speed range is. Some jet fighters for instance are not designed for sustained supersonic flight and therefore can use quite a simple engine diffuser.

## Convergent-Divergent Diffuser

### Convergent - Divergent Diffuser / Inlet



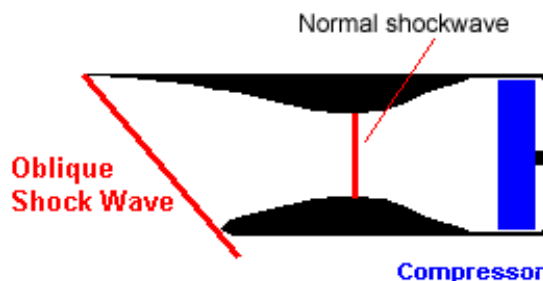
Supersonic flow entering the diffuser is compressed and slowed. In the throat it reaches the speed of sound and a Normal shockwave develops

Behind the shockwave subsonic flow expands and slows further.

Figure 153 shows a simple convergent-divergent diffuser. This design works because supersonic flow slows down when it enters a constricted (convergent) area. If the geometry of the constriction is correct the air flow reaches the speed of sound in the throat, where a Normal shockwave forms. Remember that airflow behind a Normal shockwave is always subsonic. The subsonic flow slows further in the expanding (divergent) section of the diffuser so that it is well below the speed of sound by the time it reaches the compressor.

Figure 153

### Convergent - Divergent Diffuser / Inlet



A variation of the Convergent-Divergent compressor.

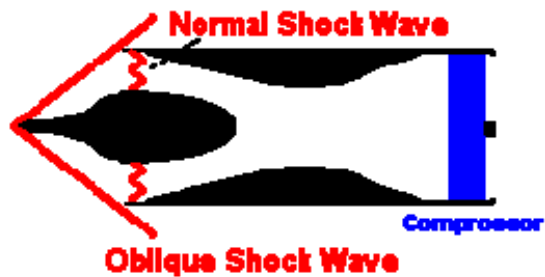
The oblique shockwave is not captured inside the inlet. It slows the air flow so the Normal shockwave is weaker and therefore wastes less energy.

Obviously the geometry of the diffuser has to be specific to the speed the aircraft is flying. Therefore, if the aircraft is to operate over a broad range of supersonic speeds a more complex system is required. The convergent-divergent diffuser is only suitable for short bursts of supersonic flight, at less than Mach 2 (such as on a Fighter.)



*You can see the convergent-divergent diffuser engine inlet of this F-15.*

## Center Body Diffuser



**The center body can be mounted on tracks. It must be moved further out as the aircraft flies faster.**

## Center-Body Diffuser

A more elaborate type of diffuser is the center-body design, which has a sharp center-body that strikes the airflow producing one or more weak oblique shockwaves and one weak Normal shockwave.

The inlet geometry is such that air is drawn into the engine inlet at right angles to Normal shockwave. The resulting flow is subsonic.

A divergent chamber then slows the airflow further before it reaches the compressor.

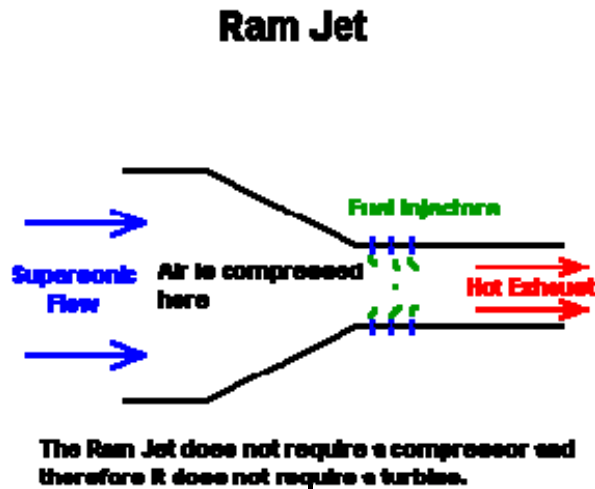
As with the convergent-divergent design, the geometry is critical (i.e. how far ahead of the inlet the center-body is.) The solution is to mount the center body on a track mechanism that automatically extends as the aircraft fly's faster, and retracts when it slows down. This design is suited to sustained supersonic flight and therefore would be a better choice for a future supersonic airliner.





*The center-body differs are clearly visible on the SR71*





### ***Ram Jets***

The Ram Jet takes advantage of the way a venturi works in supersonic flow. The convergent section of a venturi compresses supersonic air. Therefore an engine compressor section is not needed, as shown in Figure 154

Since the turbine section of a jet engine is **ONLY** needed to drive the compressor, the turbines can be eliminated too. The result is a very simple and efficient jet engine known as a Ram Jet.

Figure 154

The problem with the Ram Jet is that it will only work in supersonic flight. Therefore, the aircraft would need a different engine for subsonic flight. Nevertheless Ram Jets may someday be used on Supersonic or Hypersonic aircraft.

## **Conclusion**

We have reached the end of the journey I invited you on. I hope you have a better appreciation for how your aeroplane flies now. My hope is that you have discovered patterns in the explanations and are able to relate all the details you have learned to each other. Aerodynamics actually depends on only a small number of basic principles. These are encoded in the six rules. Everything else follows logically from them.

You may need to read this text several times before you fully see the connections between the different topics. But, once you do see them your understanding will be much deeper.

I truly believe that you can fly better, by which I mean more precisely with less mental and physical effort, if you understand aerodynamics. I wish you safe and happy flights for the rest of your aviation career.